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TIDAL FLOW IN ENTRANCES; WATER-LEVEL FLUCTUATIONS OF BASINS IN COMMUNICATION WITH SEAS

G. H. Keulegan

Committee on Tidal Hydraulics (Army)
Washington, D. C.

July 1967

TECHNICAL BULLETIN NO. 14

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REPORTS OF COMMITTEE ON TIDAL HYDRAULICS

Report No.	Title	Date
1	Evaluation of Present State of Knowledge of Factors Affecting Tidal Hydraulics and Related Phenomena	Feb. 1950
2	Bibliography on Tidal Hydraulics	Feb. 1954
	Supplement No. 1, Material Compiled Through May 1955	June 1955
	Supplement No. 2, Material Compiled from May 1955 to May 1957	May 1957
	Supplement No. 3, Material Compiled from May 1957 to May 1959	May 1959
	Supplement No. 4, Material Compiled from May 1959 to May 1965	May 1965
3	Evaluation of Present State of Knowledge of Factors Affecting Tidal Hydraulics and Related Phenomena (revised edition of Report No. 1)	May 1965

Technical Bulletin No.	Title	Date
1	Sediment Discharge Measurements in Tidal Waterways	May 1954
2	Fresh Water-Salt Water Density Currents, a Major Cause of Siltation in Estuaries	April 1957
3	Tidal Flow in Entrances	Jan. 1960
4	Soil as a Factor in Shoaling Processes, a Literature Review	June 1960
5	One-Dimensional Analysis of Salinity Intrusion in Estuaries	June 1961
6	Typical Major Tidal Hydraulic Problems in United States and Research Sponsored by the Corps of Engineers Committee on Tidal Hydraulics	June 1963
7	A Study of Rheologic Properties of Estuarial Sediments	Sept. 1963
8	Channel Depth as a Factor in Estuarine Sedimentation	Mar. 1965
9	A Comparison of an Estuary Tide Calculation by Hydraulic Model and Computer	June 1965
10	Significance of Clay Minerals in Shoaling Problems	Sept. 1966
11	Extracts from The Manual of Tides	Sept. 1966
12	Unpublished Consultation Reports on Corps of Engineers Tidal Projects	Dec. 1966
13	Two-Dimensional Aspects of Salinity Intrusion in Estuaries: Analysis of Salinity and Velocity Distributions	June 1967
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CORPS OF ENGINEERS, U. S. ARMY

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FOREWORD

This report describes an analytical approach to the problem of defining changes in the water level of a basin connected to the ocean by a channel or channels. To illustrate use of the analytical approach, the case of Indian River, Delaware, is examined in considerable detail.

The analytical method was developed and the report was prepared by Dr. Garbis H. Keulegan. The work was initiated while Dr. Keulegan was employed as Physicist for the National Bureau of Standards and was completed while he was serving as Staff Consultant to the Hydraulics Division, U. S. Army Engineer Waterways Experiment Station.

The work reported herein was sponsored by the U. S. Army Engineer Committee on Tidal Hydraulics.

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GLOSSARY

Part I

a	Cross-sectional area of inlet water
a_1, a_3, b_3	Constants in the expansion of z, equation 32
A	Surficial area of basin
C	A dimensionless number relating Ω to Q_m, equation 56
C_d	Coefficient of discharge of barrier cuts
g	Constant of gravity
h_1	Dimensionless ratio H_1/H
h_{1m}	Dimensionless ratio H_{1m}/H
h_2	Dimensionless ratio H_2/H
H	Semirange of tide in sea
H_1	Elevation of basin water referred to msl
H_{1m}	Semirange of tide in basin
H_2	Elevation of sea water referred to msl
K	Coefficient of repletion, equation 12
K_1	Coefficient of repletion for barrier cuts, equation 67
L	Length of inlet
m	Coefficient resulting from velocity distribution, equation 2
n	Manning's roughness, equation 15
N_1, N_3, N_5	Constants in the expansion of $\sqrt{\sin \theta}$, equation 27
Q	Instantaneous discharge, rate of discharge
Q_m	Maximum rate of discharge
r	Hydraulic radius of inlet cross section
s	Surface slope
t	Time, measured from the instant when the waters of sea and of basin are at the same level and waters of sea are rising

T Period of tide, 12.42 hr

V Velocity in inlet channel

V_m Maximum mean velocity in inlet channel

z Difference in h_1 and h_2 , $z = h_2 - h_1$

α Lag of tide maxima behind sea high-water tide

β A quantity defined by equation 20

θ Specific tidal time, $2\pi t/T$

λ Coefficient of friction, equation 3

τ Specific time when sea is at msl

Ω Volume of tidal prism, $\Omega = 2H_{1m}A$

Part II

a_o Cross section of "The Ditches"

D_o Depth of undisturbed water

H_{1m}, H_{om} Semirange of tide in Indian River Bay and Rehoboth Bay, respectively

k Dimensionless constant appearing in equations 70 and 71

M_1 Factor of proportionality in equation 84

Q_i Volume of inflow during ebb of a tidal cycle

Q_o Volume of outflow during flood of a tidal cycle

Q_r Volume of river discharge into bays during a tidal cycle

Q_T Net total outflow during a tidal cycle

r Average depth in inlet between bridge and ocean

t Time, measured from instant when sea tide is crossing msl and is increasing

t_i Duration of inflow

t_1 The instant when the waters of bay and sea are in the same horizontal plane

V Wind velocity

V_o Mean velocity of the currents in "The Ditches"

$\alpha_1, \beta_1, \delta, \tau_1$ Dimensionless parameters appearing in equation 82

Δ Superelevation of the mean level of bay waters above msl

ζ, η, κ Dimensionless parameters of wind tide equation, equation 95

μ Dimensionless constant proportional to k , equation 78

ρ	Density of water
ρ_a	Density of air
σ	$2\pi/T$
τ_s	Wind stress
χ	Taylor stress coefficient

ABSTRACT

An analysis is made of the changes in water level in a basin connected to the ocean by a relatively long channel. It is assumed that the banks of the basin are vertical, that the connecting channel is many times deeper than the tidal range, that there is no flow into the basin from streams, and that, therefore, there are no density currents present. The consideration of the storage equation shows that the coefficient of repletion is a characteristic quantity for basins in general. The aim of the analysis is to determine the range of tides within the basin, the lag between the maximum surface deflections in the basin and in the sea, and the maximum mean velocity in the connecting channel. These quantities are determined as functions of the coefficient of repletion. Tables are given for ready reference for many of the quantities.

The application of the method for channels to flow through barrier cuts is also discussed. Finally, the matters of the Indian River Inlet are studied in detail to illustrate the practical meaning of the various results of the main analysis.

TIDAL FLOW IN ENTRANCES

WATER-LEVEL FLUCTUATIONS OF BASINS IN COMMUNICATION WITH SEAS

PART I: THEORETICAL DEVELOPMENTS

Introduction

The study of the water-level fluctuations in basins communicating with seas can be undertaken with reference to a number of conditions, namely, shape (horizontal projections) of the basin, slopes of the banks, amount of inflow from streams, number of connections with the sea, hydraulic resistance of the connecting channels, type of tidal fluctuations of the seas, relation between tidal range and depth of channel, and presence or absence of density currents.

The movement of water in the channel in the horizontal direction is affected in a very marked manner by the shape of the basin. If the basin is narrow, shallow, and long and the communication with the sea is at one end, the flow of water from the sea into the basin may be associated with advancing waves. In this case, change of water level in the basin is not uniform; hence the various tidal changes in the basin from place to place must be considered. On the other hand, if the basin is square or circular, or of some similar form, the change in elevation of the water surface will be nearly the same for every point in the basin.

In the latter case, the accumulation of water in the basin obeys the condition of the flow of reservoirs and, in particular, the so-called storage law. The equation representing the storage is also the differential equation of the surface changes. The form of the equation and hence the method of solving it are affected by inflow from streams, slope of the banks, ratio of the depth of the connection channel to the tidal range, and way in which flood and ebb tides occur.

This report discusses the simplest set of conditions. It is assumed here that the walls of the basin are vertical, that there is no inflow from streams, that there are no density currents present, and that the tidal fluctuations are given by a sine curve. The connecting channel is assumed

to be prismatic, and the depth of the channel is assumed to be large with respect to the tidal range. It is also assumed that the flow in the channel is governed by Manning's formula.

Although this problem has been considered by COL Earl I. Brown, who has given a solution,[1] it was believed that a new treatment should be attempted with the object of establishing a better approximation. A novelty in the solution presented here is the dimensionless form of the equation of the surface changes. The aim of the analysis is to evaluate first the maximum displacement of the water surface in the basin, and second the maximum mean velocity in the channel during a tidal cycle.

Derivation of the Equation of Surface Changes

Let Q be the discharge at time t in the connecting channel, V the mean velocity, H_2 the elevation of the open sea, and H_1 the elevation of the water surface in the basin. The elevations are all measured with respect to mean sea level (msl). The difference, $H_2 - H_1$, represents the fall of the surface corresponding to the mean velocity V (fig. 1).

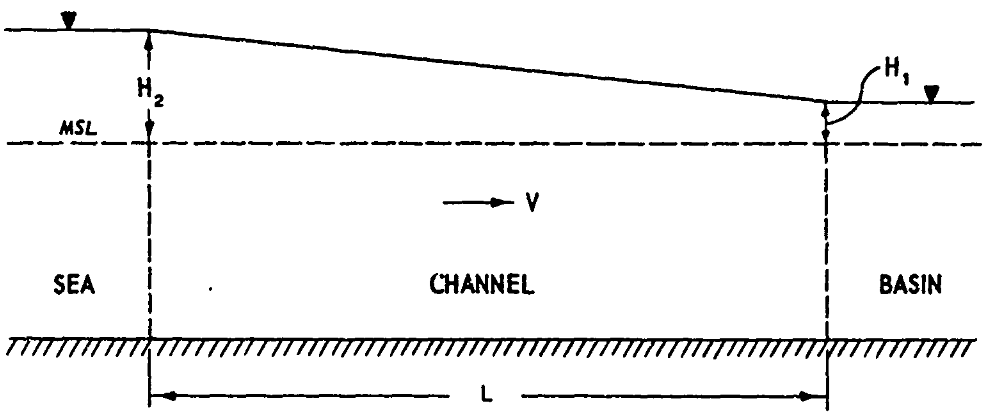

Fig. 1. Gradient of water surface over channel

This fall can be broken into two parts as follows:

$$H_2 - H_1 = \Delta H_1 + \Delta H_2 \tag{1}$$

The first difference gives the fall that is necessary to accelerate the water from the sea entering the channel from zero to the velocity V at the entrance. Accordingly,

$$\Delta H_1 = m \frac{V^2}{2g} \tag{2}$$

where m is a coefficient resulting from the velocity distribution and g is the acceleration due to gravity. If the velocity distribution over the cross section is uniform, m reduces to unity. In open-channel flow the exact value of m is not known.

Since the jet issuing from the exit end of the channel is dissipated in a process of turbulent expansion and the pressures in the medium where the jet is being obliterated are hydrostatic, the difference ΔH_2 gives the fall necessary to overcome the resistance of the connecting channel. Using the Weisbach type of formula:

$$\Delta H_2 = \lambda \frac{L}{r} \frac{V^2}{2g} \tag{3}$$

where λ is the friction coefficient, L is the length of the connecting channel, and r is the hydraulic radius of the channel. The relation that exists between λ and Manning's n will be considered later.

Combining the expressions in equations 2 and 3:

$$\Delta H_1 + \Delta H_2 = \left(\lambda \frac{L}{r} + m\right) \frac{V^2}{2g} \tag{4}$$

Introducing the difference, $H_2 - H_1$, and solving for V^2, yields:

$$V^2 = \frac{2gr}{\lambda L + mr} (H_2 - H_1) \tag{5}$$

It is desirable to express the [illegible] ations with respect to msl in terms of the semiamplitude of the tidal displacements occurring in the sea. Denoting the tidal range by $2H$, and using H as a measure of the surface fluctuations, yields:

$$V^2 = \frac{2grH}{\lambda L + mr}\left(\frac{H_2}{H} - \frac{H_1}{H}\right)$$

or

$$V = \sqrt{\frac{2grH}{\lambda L + mr}}\sqrt{\frac{H_2}{H} - \frac{H_1}{H}} \qquad (6)$$

Writing $h_2 = H_2/H$, and $h_1 = H_1/H$, then

$$V = \sqrt{\frac{2grH}{\lambda L + mr}}\sqrt{h_2 - h_1} \qquad (7)$$

Since there are no contributions from the inflow of streams and since the banks are vertical, the storage equation of the water in the basin, assuming that the tide rises and falls simultaneously throughout the basin, is

$$A\frac{dH_1}{dt} = aV \qquad (8)$$

where A is the surficial area of the basin, and a is the cross-sectional area of the connecting prismatic channel. Hence,

$$\frac{dH_1}{dt} = \frac{a}{A}V \qquad (9)$$

Denoting the tidal period by T permits introduction of the transformation

$$\frac{t}{T} = \frac{\theta}{2\pi} \qquad (10)$$

where θ is specific tidal time in radians, and equation 9 becomes

$$\frac{dh_1}{d\theta} = \frac{T}{2\pi H}\frac{a}{A}V \qquad (11)$$

Eliminating V between equations 7 and 11 and writing

$$K = \frac{T}{2\pi H}\frac{a}{A}\sqrt{\frac{2grH}{\lambda L + mr}} \qquad (12)$$

produces finally

$$\frac{dh_1}{d\theta} = K\sqrt{h_2 - h_1} \ , \ h_2 > h_1 \tag{13}$$

which is the differential equation of the surface fluctuations in the basin when the surface of the sea is at a higher elevation than the surface of the water in the basin. When the condition is reversed, i.e. when the surface of the sea is lower than the surface of the water in the basin, the corresponding equation is

$$\frac{dh_1}{d\theta} = -K\sqrt{h_1 - h_2} \ , \ h_1 > h_2 \tag{14}$$

In the results that will be given later, the numerical quantity K plays a decisive role. It summarizes the effects of the channel and the basin dimensions, of the roughness of the walls, and of the period and range of the tidal fluctuations on the limits of the water-level changes in the basin. Because of this significance it appears to be appropriate to refer to K as the coefficient of filling or repletion.

Typical Values of the Coefficient of Repletion

It is desired to consider some typical values of the coefficient K in order to form an idea of the variations in the values of the coefficient which are ordinarily to be expected. The first step is to give the relation that exists between the coefficient of friction λ and Manning's n. Manning's formula is

$$V = \frac{1.486}{n} r^{1/6} \sqrt{rs}$$

where s, the surface slope, is equal to $\Delta H_2/L$, and the units of measurement are the foot and the second. But from equation 3

$$V = \sqrt{\frac{2g}{\lambda} rs}$$

Comparing the latter two expressions for the velocity yields

$$\sqrt{\lambda} = \frac{n\sqrt{2g}}{1.486r^{1/6}} \tag{15}$$

which is the desired relation connecting λ with n . Some numerical values relating λ to n and r are given in table 1.

If it is assumed that the distribution of the velocities at the entrance to the channel is uniform, m in equation 12 can be taken as unity. The relation giving K can now be written as

$$\frac{AK\sqrt{H}}{a} = \frac{T\sqrt{2g}}{2\pi}\sqrt{\frac{r}{\lambda L + r}} \tag{12a}$$

The quantity on the right side of the equation has been computed for various channel depths and lengths and for different values of n . Computations are made for an assumed tidal period T equal to 12 hr. All lengths are in feet and times in seconds in the computations. For the results see table 2. As an illustration of the use of the table, assume that $r = 5$ ft, $L = 1000$ ft, $n = 0.03$, and $H = 1$ ft. Then from table 2:

$$K = 2.734 \times 10^4 \times \frac{a}{A}$$

so that if A/a is given the succession of values, 10^3, 10^4, and 10^5, K takes on the values, 27.3, 2.73, and 0.27, respectively.

Sinusoidal Fluctuations of Surface of Sea

Granting that the fluctuations in level of the surface of the sea can be represented as a pure sine curve, the most general solution to describe the fluctuations of water-surface level in the basin needs to be effected with reference to the height of the water surface in the basin at the instant the basin is connected to the sea.

If, however, the basin has been in communication with the sea for a long time, the fluctuations of the water surface in the basin become steady and fluctuate between limits that do not vary with time. The same limits

are established no matter what the initial depth of water in the basin may have been. Although the fluctuations of the surface of the water in the basin are periodic, it cannot be said that they have the form of a pure sine curve, because the frictional resistance of the connecting channel varies as the square of the mean velocity.

Assume that the displacements of the water surfaces in the sea and in the basin are given on a common axis of time t or of the dimensionless time parameter θ (see fig. 2). The origin of time may be taken as the

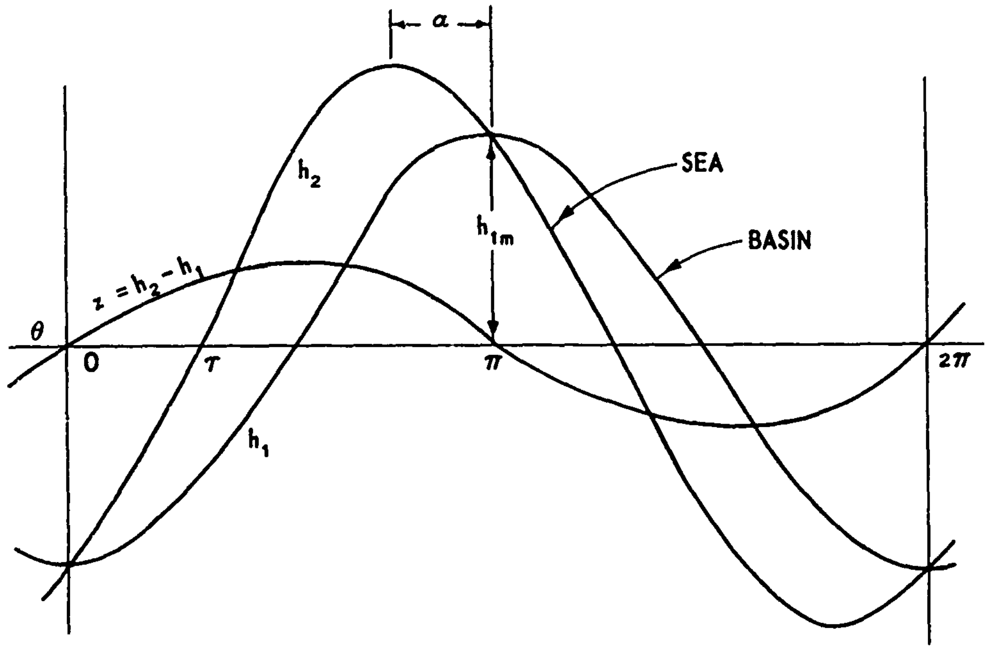

Fig. 2. Surface fluctuations of sea and basin

instant when h_2 and h_1 are equal and h_2 commences to become greater than h_1. Then, as shown in fig. 2, $h_2 = 0$ when $\theta = \tau$. Accordingly, the oscillation of the surface of the sea will be given by

$$h_2 = \sin(\theta - \tau) \,, \; 0 < \theta < 2\pi \tag{16}$$

A consideration that will have a bearing on the method of solution to be followed is the proportion of the time during which the water level in the sea is higher than the water level in the basin. It will be assumed that during the period $T/2$ the surface of the sea is higher than the surface

in the basin. The assumption will be verified later. During the following period the reverse is true. Accordingly, the determination of h_1 will be made separately for the ranges of values $0 < \theta < \pi$ and $\pi < \theta < 2\pi$; these ranges will be referred to as the first range and the second range, respectively.

The mathematical task involves the following relations for the first range:

$$h_2 > h_1 \ , \ 0 < \theta < \pi$$

$$h_2 = \sin (\theta - \tau) \tag{16}$$

$$\frac{dh_1}{d\theta} = K\sqrt{h_2 - h_1} \tag{13}$$

$$h_2 = h_1 \ , \ \theta = 0$$

and

$$h_2 = h_1 \ , \ \theta = \pi$$

For the second range, the relations involved are:

$$h_1 > h_2 \ , \ \pi < \theta < 2\pi$$

$$h_2 = \sin (\theta - \tau) \tag{16}$$

$$\frac{dh_1}{d\theta} = -K\sqrt{h_1 - h_2} \tag{14}$$

$$h_2 = h_1 \ , \ \theta = \pi$$

and

$$h_2 = h_1 \ , \ \theta = 2\pi$$

Instead of determining h_1 directly, it is more convenient to obtain the difference, $h_1 - h_2$. Thus, putting

$$z = h_2 - h_1 \ , \ 0 < \theta < \pi \tag{17}$$

equation 13 reduces to

$$\frac{dz}{d\theta} = -K\sqrt{z} + \frac{dh_2}{d\theta}$$

After introducing the value of h_2 from equation 16. the mathematical problem for the first range becomes that of determining z from the following relations:

$$z > 0 \ , \ 0 < \theta < \pi$$

$$\frac{dz}{d\theta} = -K\sqrt{z} + \cos\theta\cos\tau + \sin\theta\sin\tau \tag{18}$$

$$z = 0 \ , \ \theta = 0$$

$$z = 0 \ , \ \theta = \pi$$

For the second range, putting

$$z = h_1 - h_2 \ , \ \pi < \theta < 2\pi \tag{19}$$

equation 14 becomes

$$\frac{dz}{d\theta} = -K\sqrt{z} - \frac{dh_2}{d\theta}$$

Introducing the transformation

$$\theta = \pi + \beta \tag{20}$$

and hence the relation

$$h_2 = -\sin(\beta - \tau)$$

the mathematical problem for the second range involves the solution of the conditions

$$z > 0 \ , \ 0 < \beta < \pi$$

$$\frac{dz}{d\beta} = -K\sqrt{z} + \cos\beta\cos\tau + \sin\beta\sin\tau \tag{21}$$

$$z = 0 \ , \ \beta = 0$$

$$z = 0 \ , \ \beta = \pi$$

Comparison of the systems of relations in equations 18 and 21 indicates that it should be sufficient to obtain the solution of z for the first range. The behavior of z for the second range is readily deduced. The same comparison shows also that the portion of the time during which the surface of the sea is at a higher level than the surface in the basin is $T/2$. Thus the original assumption is confirmed.

Now since the solution of equation 18 is of the form

$$z = f(\theta) \tag{22}$$

and since

$$h_1 = h_2 - z \ , \ 0 < \theta < \pi$$

the value of h_1 for this range is

$$h_1 = \sin(\theta - \tau) - f(\theta) \tag{23}$$

Again, since the solution of equation 21 is of the form

$$z = f(\beta) \tag{24}$$

and since

$$h_1 = z + h_2 \ , \ 0 < \beta < \pi$$

the value of h_1 for the second range is

$$h_1 = \sin(\beta - \tau) + f(\beta) \; , \; \theta = \pi + \beta \tag{25}$$

The geometrical interpretation of the above discussion is readily inferred. The curve of h_1 for the first range is drawn. The curve is reflected and is moved along the axis of θ by an amount π. The curve thus displaced is the curve of h_1 for the second range.

Fourier Expansion of $\sqrt{\sin \theta}$

The determination of a form of z that will satisfy the differential equation, equation 18, can be effected in various ways. The solution can be in terms of polynomials of θ or in terms of the circular functions of θ. Since periodic changes are involved, it is preferable to obtain the solution in circular functions. Select as a possible expression the series

$$z = \sum_1^{\infty} A_n \sin n\theta + \sum_1^{\infty} B_n[\cos n\theta - \cos(n+2)\theta] \tag{26}$$

$$n = 1, 3, 5, \ldots, 2m + 1$$

since z vanishes for $\theta = 0$ and $\theta = \pi$. Here, A and B are constants.

It is obvious that the first term in the series, i.e. the quantity $A_1 \sin \theta$, is the predominant term. In equation 18 there is the term $\sqrt{z}$, and it will be necessary in the course of the analysis, as will be seen later, to have the Fourier expansion of $\sqrt{\sin \theta}$. Now, by the rule of the Fourier expansion, if $f_1(\theta)$ is single-valued, finite, and continuous between $\theta = 0$ and $\theta = \pi$, it can be developed into a series of the form[2]

$$f_1(\theta) = a_1 \sin \theta + a_2 \sin 2\theta + a_3 \sin 3\theta$$

where the coefficients have the values

$$a_m = \frac{2}{\pi}\int_0^\pi f_1(\theta)\ \sin m\theta\ d\theta$$

In conformity with the rule and by inspection, the following equation can be written:

$$\sqrt{\sin\theta} = N_1 \sin\theta + N_3 \sin 3\theta + N_5 \sin 5\theta \tag{27}$$

where

$$\left.\begin{aligned} N_1 &= \frac{2}{\pi}\int_0^\pi \sqrt{\sin\theta}\ \sin\theta\ d\theta \\ N_3 &= \frac{2}{\pi}\int_0^\pi \sqrt{\sin\theta}\ \sin 3\theta\ d\theta \\ N_5 &= \frac{2}{\pi}\int_0^\pi \sqrt{\sin\theta}\ \sin 5\theta\ d\theta \end{aligned}\right\} \tag{28}$$

The N's can be determined numerically by replacing the process of integration by a process of summation. For example:

$$N_1 = \frac{2}{\pi}\sum \sqrt{\sin\theta}\ \sin\theta\ \Delta\theta\ ,\ 0 \leq \theta \leq \pi$$

The summations are made by putting the interval $\Delta\theta$ equal to 0.03491 radians. The computations give

$$\left.\begin{aligned} N_1 &= 1.1107 \\ N_3 &= 0.1580 \\ N_5 &= 0.0711 \end{aligned}\right\} \tag{29}$$

The degree of convergence of the series in equation 27 will be examined best when the differences

$$\Delta_1 = \sqrt{\sin\theta} - N_1 \sin\theta$$

$$\Delta_3 = \sqrt{\sin\theta} - N_1 \sin\theta - N_3 \sin 3\theta$$

$$\Delta_5 = \sqrt{\sin\theta} - N_1 \sin\theta - N_3 \sin 3\theta - N_5 \sin 5\theta$$

are computed and plotted (see fig. 3). The convergence appears to be satisfactory. The larger deviations are confined to the end regions. The effect of adding sine terms of the higher frequencies is to decrease the interval length of the end regions where the maximum deviations occur. In

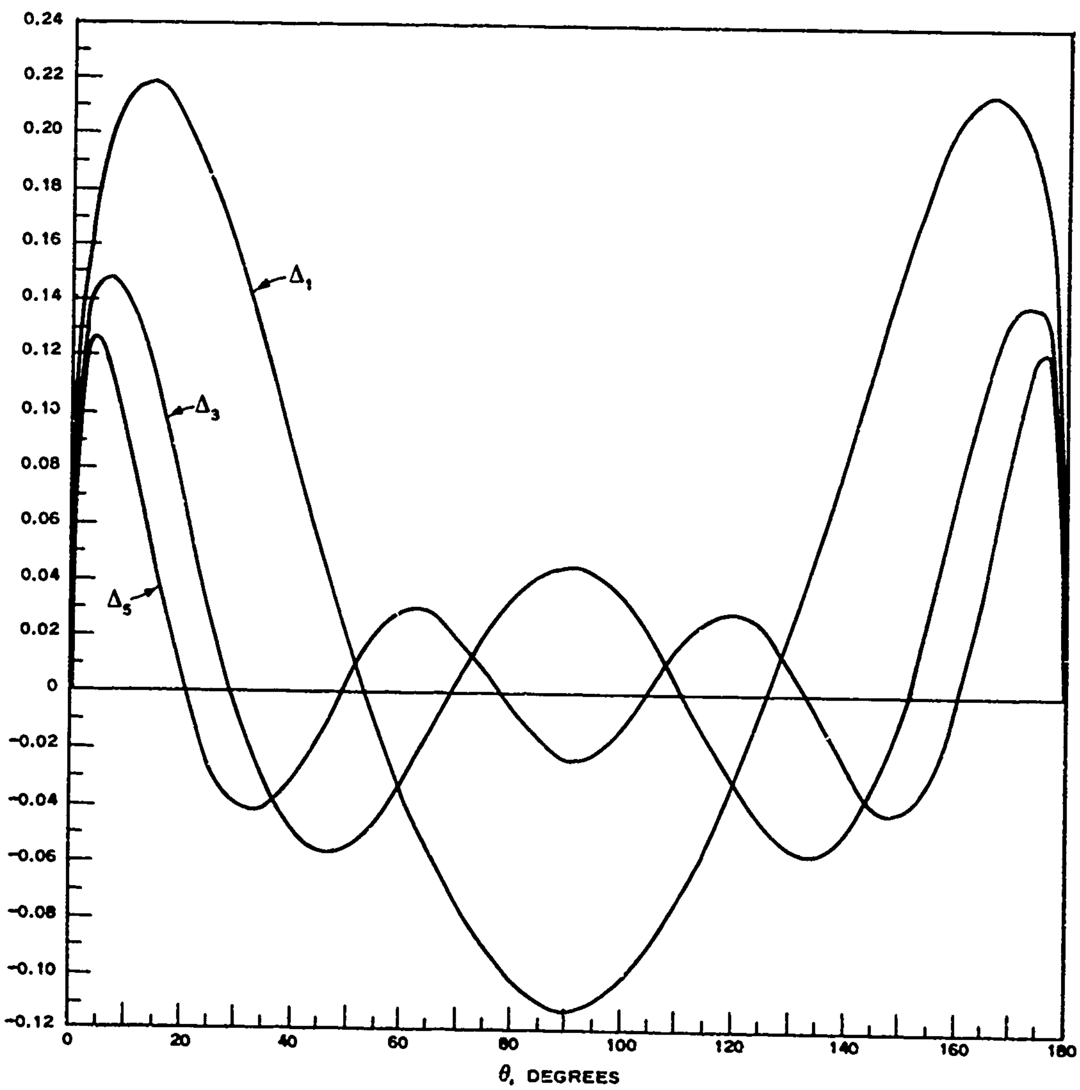

Fig. 3. Residuals in the successive approximations of the Fourier expansion of $\sqrt{\sin\theta}$

addition, the magnitudes of the deviations are decreased. It appears also that for the ordinary approximations, when very high accuracy is not needed, it is sufficient to adopt:

$$\sqrt{\sin\theta} = N_1 \sin\theta + N_3 \sin 3\theta \tag{30}$$

Conforming to this selection, the terms of higher frequencies in the expression for z, equation 26, could be dispensed with.

Method of Approximate Solutions

As was mentioned above, it will be sufficient to determine z for the first range only, i.e. for the interval $0 < \theta < \pi$. The differential equation to be solved is

$$\frac{dz}{d\theta} = -K\sqrt{z} + \cos\theta\cos\tau + \sin\theta\sin\tau \tag{18}$$

The solution is taken to be of the form

$$z = a_1 \sin\theta + a_1 b_3(\cos\theta - \cos 3\theta) + a_1 a_3 \sin 3\theta \tag{31}$$

an expression which vanishes for the points, $\theta = 0$ and $\theta = \tau$. Since terms of higher frequency are omitted, the solution represents an approximation. The unknown quantities are the coefficients, a_1, a_3, b_3, and the phase angle τ.

Factoring out the first term on the right side of equation 31 yields

$$z = a_1 \sin\theta\left(1 + b_3\,\frac{\cos\theta - \cos 3\theta}{\sin\theta} + a_3\,\frac{\sin 3\theta}{\sin\theta}\right) \tag{32}$$

Since the terms in parentheses remain finite when θ is varied from 0 to π, it is admissible to take the square root of both sides. Hence,

$$z^{1/2} = a_1^{1/2}\sqrt{\sin\theta}\left(1 + \frac{b_3}{2}\,\frac{\cos\theta - \cos 3\theta}{\sin\theta} + \frac{a_3}{2}\,\frac{\sin 3\theta}{\sin\theta}\right) \tag{33}$$

It is conceived that a_3 and b_3 are fractions; therefore, in writing the square-root expression for the terms in the parentheses of equation 32, the terms containing the squares of a_3 and b_3 and their product are neglected. Introducing the Fourier expression of equation 30 yields

$$z^{1/2} = a_1^{1/2}(N_1 \sin \theta + N_3 \sin 3\theta)\left(1 + \frac{1}{2}b_3 \frac{\cos \theta - \cos 3\theta}{\sin \theta} + \frac{1}{2}a_3 \frac{\sin 3\theta}{\sin \theta}\right) \tag{34}$$

Removing the first set of parentheses and ignoring the terms multiplied by N_3b_3 and N_3a_3 , since these are small fractions, yields

$$z^{1/2} = a_1^{1/2}\left[N_1 \sin \theta + N_3 \sin 3\theta + \frac{b_3N_1}{2}(\cos \theta - \cos 3\theta) + \frac{a_3N_1}{2} \sin 3\theta\right] \tag{35}$$

Differentiating equation 31 yields

$$\frac{dz}{d\theta} = a_1 \cos \theta + a_1b_3(-\sin \theta + 3 \sin 3\theta) + 3a_1a_3 \cos 3\theta \tag{36}$$

Substituting the expressions from equations 35 and 36 in equation 18, the latter equation reduces to

$$\left(a_1^{1/2} N_1K - a_1b_3 - \sin \tau\right)\sin \theta + \left(a_1^{1/2} \frac{N_1K}{2} b_3 + a_1 - \cos \tau\right) \cos \theta + \left[a_1^{1/2} K \left(N_3 + \frac{N_1}{2} a_3\right) + 3a_1b_3\right] \sin 3\theta + \left(-a_1^{1/2} \frac{N_1K}{2} b_3 + 3a_1a_3\right)\cos 3\theta = 0 \tag{37}$$

This equation must hold for any value of θ . Hence it must hold when multiplied by $\sin \theta\, d\theta$ and integrated between the limits 0 and π . The same is true when it is multiplied by $\cos \theta\, d\theta$, $\cos 3\theta\, d\theta$, or $\sin 3\theta\, d\theta$ and integrated between the same limits. Carrying out all the steps, it is found that

$$a_1^{1/2} N_1 K - a_1 b_3 - \sin\tau = 0 \tag{38}$$

$$a_1^{1/2} \frac{N_1 K}{2} b_3 + a_1 - \cos\tau = 0 \tag{39}$$

$$a_1^{1/2} K \left(N_3 + \frac{N_1 a_3}{2}\right) + 3a_1 b_3 = 0 \tag{40}$$

and

$$-a_1^{1/2} \frac{N_1 K}{2} b_3 + 3a_1 a_3 = 0 \tag{41}$$

and these equations are sufficient to determine the unknowns, τ , a_1 , a_3 , and b_3 .

Obviously the rigorous determination of the unknowns is involved. Since a_3 and b_3 are small quantities, the values obtained by the method of approximations will be satisfactory. Discarding in equations 38 and 39 the terms multiplied by b_3 , these equations simplify to

$$a_1^{1/2} N_1 K = \sin\tau \tag{42}$$

and

$$a_1 = \cos\tau \tag{43}$$

Squaring and adding gives

$$a_1^2 + a_1 N_1^2 K^2 = 1 \tag{44}$$

Solving for a_1 and denoting the positive root by a_{11} :

$$a_{11} = \sqrt{1 + \frac{N_1^4 K^4}{4}} - \frac{N_1^2 K^2}{2} \tag{45}$$

The corresponding solution for τ is

$$a_{11} = \cos\tau_1 \tag{46}$$

The quantities a_{11} and τ_1 are referred to as the first approximate values of a_1 and τ .

Substituting a_{11} for a_1 in equations 40 and 41 and solving yields

$$a_3 = -2 \frac{N_3}{N_1} \left(\frac{N_1^2 K^2}{N_1^2 K^2 + 36 a_{11}} \right) \tag{47}$$

and

$$b_3 = \frac{6 a_{11}^{1/2}}{N_1 K} a_3 \tag{48}$$

With the values of a_3 and b_3 thus known, the second approximate values of a_1 and τ can be obtained by reverting once more to equations 38 and 39. Writing

$$a_1 = a_{11} + \delta a_1 \tag{49}$$

and

$$\tau = \tau_1 + \delta\tau \tag{50}$$

it is found that:

$$\delta a_1 = \frac{a_{11}^{3/2} N_1 K}{2a_{11} + N_1^2 K^2} b_3 \tag{51}$$

and

$$\delta\tau = -\left(\frac{1}{2} + \frac{a_{11}}{2a_{11} + N_1^2 K^2} \right) b_3 \tag{52}$$

Bearing in mind that the fluctuations of the water surface in the basin are given by $h_1 = z + h_2$ and that z is given by equation 31, the quantities a_1, a_3, b_3, and τ are the parameters that determine the form of the fluctuations in the basin as a function of time. These constants depend individually on the coefficient of repletion K. Their values, determined according to the scheme of relations discussed above, are given in table 3.

In the method used, the results are obtained within the solution to a second approximation of the differential equation, equation 18. The method of analysis is such that one can go to higher approximations. This possibility, however, has only a theoretical significance, since the computations to be made are very long and should be avoided.

Surface Displacement Curves for the Case when K is Equal to Unity

By way of illustration, the surface curves for the case in which the coefficient of repletion, K of equation 12, is equal to unity will be determined. Table 3 shows that in this case the surface curve parameters are:

$$a_1 = 0.5451 \qquad \cos\tau = 0.5178$$
$$a_3 = -0.0165 \qquad \sin\tau = 0.8555$$
$$b_3 = -0.0664$$

Substituting these values in equations 16 and 31, the quantities h_2 and z are determined. Since $z = h_1 - h_2$, h_1 is computed by taking the sum of z and h_2. The results are shown in fig. 4. In preparing the

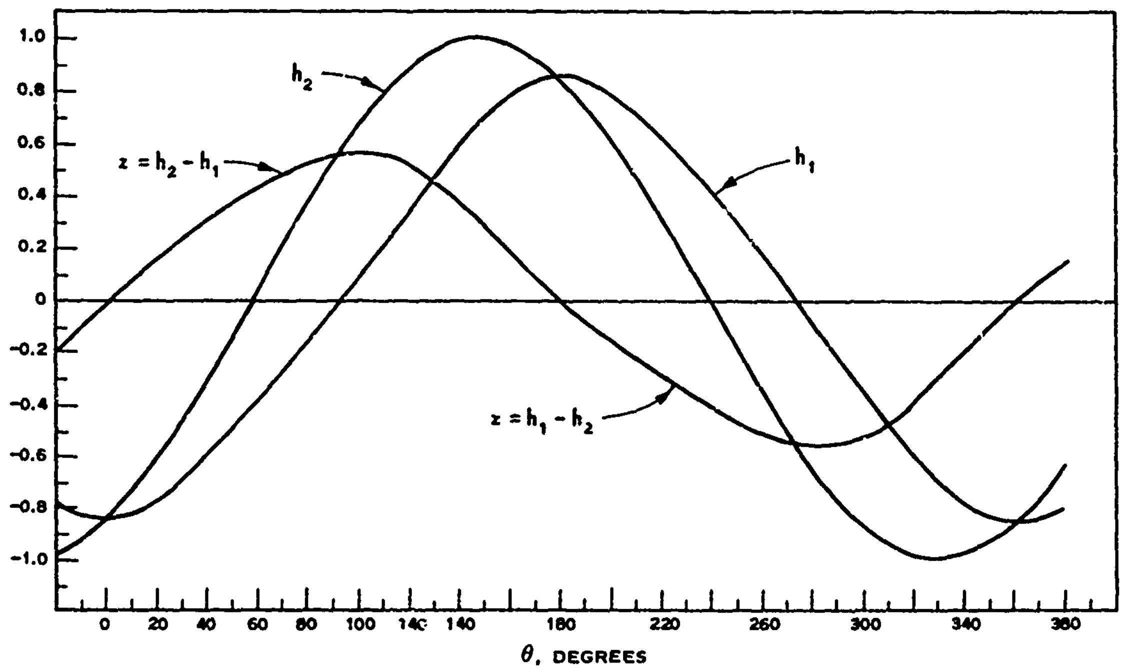

Fig. 4. Surface fluctuations of sea and basin waters for the case $K = 1$

plots, use was made of the fact that the solutions for the range $0 < \theta < \pi$ can be extended to the range $\pi < \theta < 2\pi$. This matter has been discussed previously.

The solutions given are only approximate, and it would be instructive to find how closely the differential equation, equation 18, is satisfied. Because of the approximations, if the derived solution is reintroduced into the differential equation, a remainder ΔR will be left; that is,

$$\Delta R = \frac{dz}{d\theta} + K\sqrt{z} - \frac{dh_2}{d\theta} \tag{53}$$

The smaller the difference ΔR, the closer the approximation will be to the exact solution. In this case an idea can be obtained concerning the sufficiency of the approximation by comparing the remainder ΔR with $dh_2/d\theta$. For this comparison the quantities ΔR and $dh_2/d\theta$ are plotted against θ in fig. 5. It can be seen that the remainder is small, and

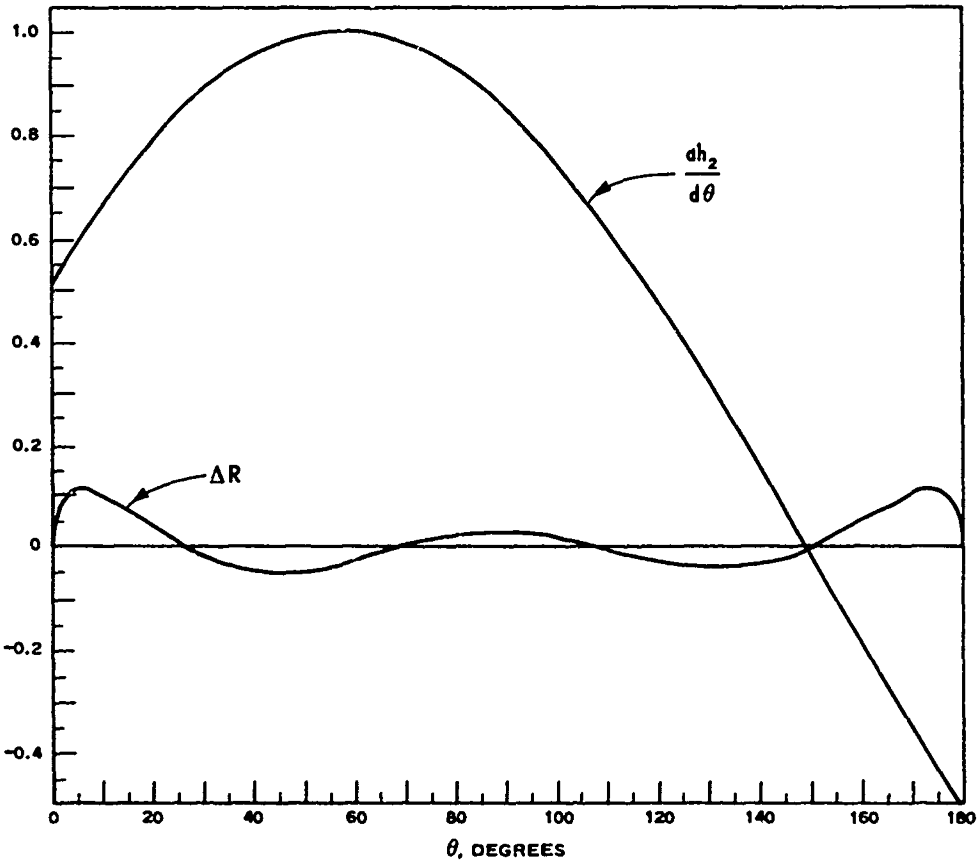

Fig. 5. Residual from the second approximate solution for the case $K = 1$

that a good solution has been obtained in the case under consideration.

Range of Tides in the Basin and Lag of the Maxima

The range of the tides in the basin is twice the maximum displacement of the water surface in the basin measured from msl. The maximum and the minimum displacements correspond to the zeroes of the difference z (i.e. $h_2 = h_1$), since for these points the rate $dh_1/d\theta$ vanished (see equation 13). As z vanishes at the points $\theta = 0$ and $\theta = \pi$, it is sufficient to consider the value of h_1 at $\theta = \pi$, where h_1 is maximum. Let this value be h_{1m}. Bearing in mind the dimensionless character of the quantity, it is seen that h_{1m} gives the ratio of the semirange of tides in the basin to the semirange of tides in the sea. Since at $\theta = \pi$, h_{1m} equals h_2, and the value of h_2 at $\theta = \pi$ is $\sin \tau$ (see equation 16), the ratio of the semirange of tide in the basin to the semirange of tide in the sea is

$$h_{1m} = \sin \tau \tag{54}$$

The ratio of the range of tide in the basin to the range in the sea is also $\sin \tau$. The values of $\sin \tau$ as a function of K are shown in table 4. Thus the tidal range of the water in the basin can be read directly from the table, once the coefficient of repletion K is known for the particular basin (see also fig. 6).

The next question to consider is the lag between the maximum displacement of the water surface of the sea and the maximum displacement of the water surface in the basin. In order of time, the former precedes the latter. Let the lag, expressed in radians, be denoted by α. The maximum displacement of the surface of the sea occurs at θ_m and has the value, from equation 16,

$$\theta_m - \tau = \frac{\pi}{2}, \quad \theta_m = \frac{\pi}{2} + \tau$$

The maximum displacement of the surface of the water in the basin occurs

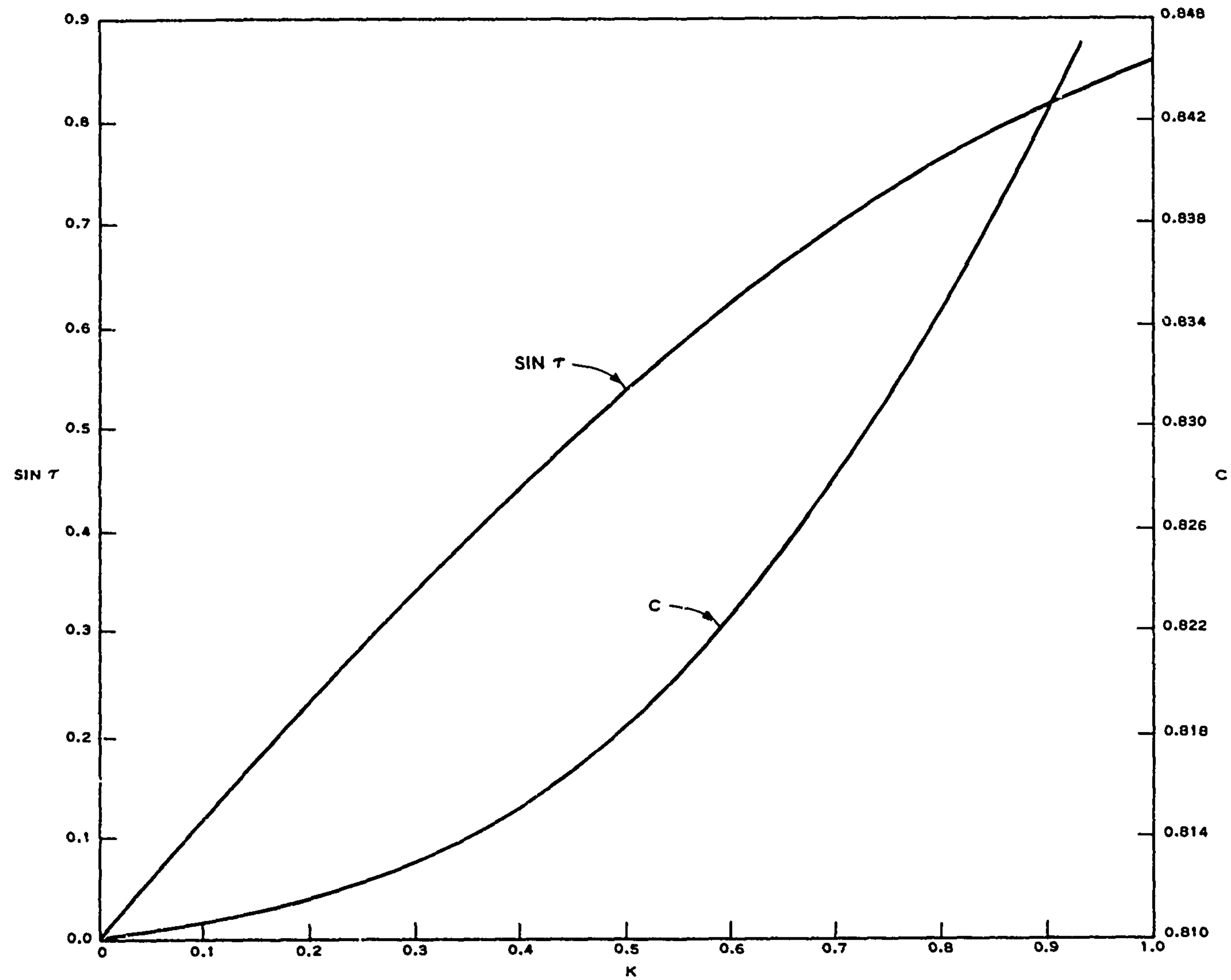

Fig. 6. Dependence of sin τ and C on coefficient of repletion K

when $\theta = \pi$. Hence the lag is:

$$\alpha = \pi - \theta_m$$

or

$$\alpha = \frac{\pi}{2} - \tau \tag{55}$$

The question of $\sin \tau$ was considered in the preceding paragraph.

Tidal Prisms and Maximum Mean Velocity in the Communicating Channel

The volume of water in the basin included between the two horizontal planes, one giving the highest elevation of the water surface during a tidal cycle and the other giving the lowest elevation of the surface, is referred to as the tidal prism. Let the volume of the prism be Ω . If Q_m is the maximum rate of discharge through the connecting channel during a half tidal cycle, the volume of the prism, the maximum rate of discharge, and the period of the tides can be connected by an expression:

$$\frac{TQ_m}{\pi\Omega} = C \tag{56}$$

where C is a dimensionless number.

The value of C is close to unity, and its exact value depends on the coefficient of repletion K . This dependence will be determined next. With the maximum mean velocity being denoted by V_m ,

$$Q_m = aV_m$$

Also, from the condition of continuity,

$$\Omega = a\int_0^{T/2} V dt$$

Since $z = h_2 - h_1$, it can be seen from equation 7 that

$$V \sim \sqrt{z}$$

and

$$V_m \sim (\sqrt{z})_m$$

the suffix m indicating that maximum values are taken. Hence,

$$\frac{Q_m}{\Omega} = \frac{(\sqrt{z})_m}{\int_0^{T/2} \sqrt{z}dt}$$

or, since $2\pi dt = Td\theta$,

$$\frac{TQ_m}{2\pi\Omega} = \frac{(\sqrt{z})_m}{\int_0^{\pi} \sqrt{z}d\theta} \tag{57}$$

Let θ_1 be the value of θ which renders z a maximum. At this point $\sqrt{z}$ is also a maximum. Accordingly, $dz/d\theta = 0$, and from equation 36:

$$\cos \theta_1 + b_3(-\sin \theta_1 + 3 \sin 3\theta_1) + 3a_3 \cos 3\theta_1 = 0$$

It can be shown that the smallest root of this equation is:

$$\theta_1 = \frac{\pi}{2} + \epsilon \tag{58}$$

where

$$\epsilon = -\frac{4}{1 - 9a_3} b_3$$

and

$$\cos \theta_1 = -\epsilon$$

$$\cos 3\theta_1 = +3\epsilon$$

$$\sin \theta_1 = 1 , \sin 3\theta_1 = -1$$

From equation 35 the maximum value of $\sqrt{z}$ is

$$(\sqrt{z})_m = a_1^{1/2}\left[N_1 \sin\theta_1 + N_3 \sin 3\theta_1 + \frac{b_3N_1}{2}(\cos\theta_1 - \cos 3\theta_1) + \frac{N_1a_3}{2}\sin 3\theta_1\right]$$

Introducing the value of θ_1 from the preceding page, this reduces to:

$$(\sqrt{z})_m = a_1^{1/2}\left(N_1 - N_3 - 2N_1b_3\epsilon - \frac{N_1a_3}{2}\right) \tag{59}$$

Again, from equation 35, effecting an integration:

$$\int_0^{\pi} z^{1/2}d\theta = 2a_1^{1/2}\left(N_1 + \frac{1}{3}N_3 + \frac{1}{6}N_1a_3\right) \tag{60}$$

Substituting these expressions, equations 59 and 60, in equation 57, and then making use of the fact that N_3, a_3, b_3, and τ are all small quantities, yields

$$\frac{TQ_m}{\pi\Omega} = 1 - \frac{4}{3}\frac{N_3}{N_1} - 2b_3\epsilon - \frac{2}{3}a_3$$

The right member of the equation is the expression for C appearing in equation 56; that is,

$$C = 1 - \frac{4}{3}\frac{N_3}{N_1} - 2b_3\epsilon - \frac{2}{3}a_3 \tag{61}$$

It is obvious that C depends on K, since a_3, b_3, and ϵ depend on K. Values of C computed using the latter expression are given in table 4. It is seen that as K is increased from 0.1 to 100 the value of C changes from 0.8106 to 1.0000 (see also fig. 6).

The formula for the tidal prism is the means by which the maximum velocity in the connecting channel can now be evaluated. By the definition of the tidal prism,

$$\Omega = 2h_{1m}AH$$

where h_{1m} is the ratio of the maximum displacement of the water surface in the basin to the maximum displacement of the surface of the sea. Hence, using equation 54

$$\Omega = 2AH \sin \tau$$

As before,

$$Q_m = V_m a$$

Introducing these into the formula for the tidal prism, equation 56, gives

$$V_m = 2\pi C \frac{A}{a} \frac{H}{T} \sin \tau \tag{62}$$

This is the relation which connects the maximum mean velocity in the connecting channel with the range of tides in the sea, $2H$. In the expression H is measured in feet and T in seconds.

Prismatic Equivalence of Irregular Channels and Multiplicity of Channels

The determination of the coefficient of repletion K of the connecting channel has been carried out, as discussed in the initial sections of this report, on the basis that the channel is prismatic. An actual channel may not meet this condition, the depth and width varying from point to point. For the purpose of evaluating K , the actual irregular channel can be replaced by a regular one having the same conductance as the irregular channel. In finding the equivalent channel it may be well to assume that the channel length L and the roughness coefficient n remain the same; also that the depth of the equivalent channel is equal to the average depth r_m of the irregular channel. Thus it remains to find the constant cross section a_s of the equivalent channel. If the end cross sections of the actual channel are not of the same size, the activity of the channel

differs for the two directions of flow, one direction being the reverse of the other. If then a_{s1} is the equivalent cross section for one direction and a_{s2} for the opposite direction, it should suffice to take the mean $(a_{s1} + a_{s2})/2$ as the cross section of the equivalent channel. If the length of the connecting channel is large in comparison with the depth, the error arising from this difficulty will be reduced.

Let r_x and a_x be the depth and the cross-sectional area of the irregular channel at the point x. Let a_m be the mean value of a_x, averaged along the entire length of the channel. Let a_1 and a_2 be the end cross sections of the channel, a_1 being on the end toward the sea. Assume that the flow Q is from the sea toward the basin (see fig. 7).

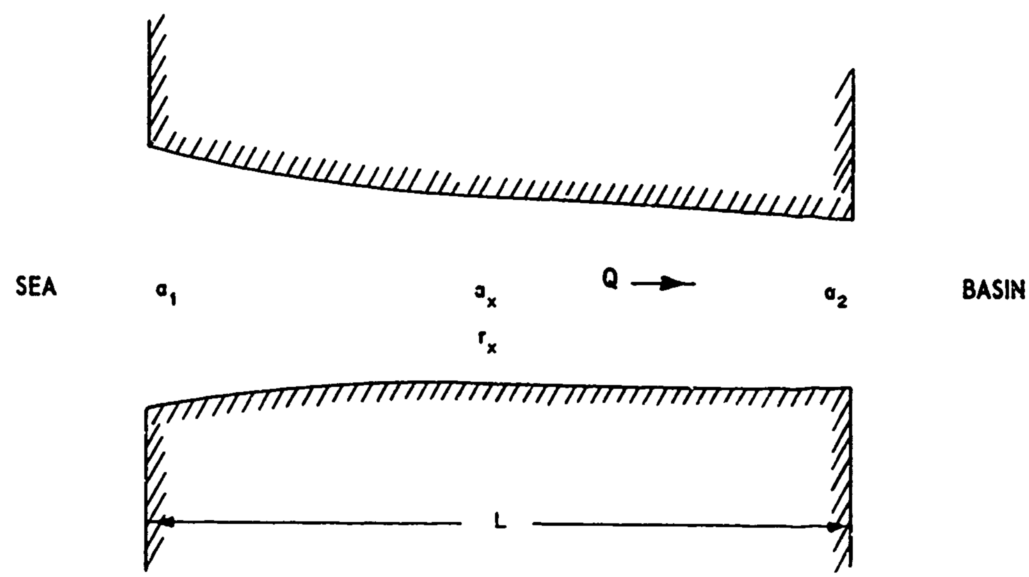

Fig. 7. An irregular connecting channel

The surface fall may be broken into two parts, ΔH_1 and ΔH_2, having the values

$$\Delta H_1 = \frac{Q^2}{2ga_1^2}$$

and

$$\Delta H_2 = \frac{Q^2}{2g}\int_0^L \frac{\lambda_x dx}{r_x a_x^2} + \frac{Q^2}{2g}\left(\frac{1}{a_2} - \frac{1}{a_1}\right)$$

Hence,

$$\Delta H = \frac{Q^2}{2g}\int_0^L \frac{\lambda_x dx}{r_x a_x^2} + \frac{Q^2}{2g}\left(\frac{1}{a_2^2}\right)$$

For the equivalent channel,

$$\Delta H = \frac{Q^2}{2g}\frac{\lambda_m L}{r_m a_s^2} + \frac{Q^2}{2g}\left(\frac{1}{a_s^2}\right)$$

In these expressions λ_x is the coefficient of friction corresponding to the hydraulic radius r_x and n , and λ_m is the coefficient of friction corresponding to the hydraulic radius r_m and n . Since channels are very wide, the average depth of a cross section can be taken equal to the hydraulic radius of the section. In channels having equal conductances, the quantity $2g\Delta H/Q^2$ is the same for the same value of Q . Applying this to the above gives

$$\frac{1}{a_s^2}\left(\frac{\lambda_m L}{r_m} + 1\right) = \int_0^L \frac{\lambda_x dx}{r_x a_x^2} + \frac{1}{a_2^2}$$

If it is assumed that along the channel the variation in depth is not very great, the above expression can be simplified. Putting $r_x = r_m + \delta r$, it can be inferred from equation 15, since δr is small with respect to r_m , that

$$\frac{\lambda_x}{r_x} = \frac{\lambda_m}{r_m}\left(1 - \frac{4}{3}\frac{\delta r}{r_m}\right)$$

Substituting this in the above equation, multiplying the resulting equation by a_m^2 , and then dividing by $\frac{\lambda_m L}{r_m} + 1$ yields

$$\frac{a_m^2}{a_s^2} = \frac{\lambda_m L}{\lambda_m L + r_m}\int_0^1 \left(1 - \frac{4}{3}\frac{\delta r}{r_m}\right)\frac{a_m^2}{a_x^2}\, d\left(\frac{x}{L}\right) + \frac{r_m}{\lambda_m L + r_m}\frac{a_m^2}{a_2^2} \qquad (63)$$

which is the formula for determining the cross section of the equivalent channel when the flow is from the sea to the basin. If a_2 is replaced by

a_1 in equation 63, the result gives the cross section of the equivalent channel for flow in the other direction.

When there is more than one connecting channel, it is a simple matter to determine the equivalent single channel. In fact, if K_1, K_2, K_3, ... are the coefficients of repletion of the individual channels, the coefficient of repletion of the single equivalent channel is

$$K = K_1 + K_2 + K_3 + \cdots \tag{64}$$

This fact will be readily understood if the derivation of equation 13 is borne in mind.

Empirical Values of the Tidal Prism Formula

Keulegan and Hall have given an empirical determination of the constant C in the tidal prism formula.[3] Based on the observed values of the actual discharges for the inlets at Nantucket, Manasquan, Beaufort, and Baker's Haulover, the average value of C is found to be 0.86.

Considering further the tidal ranges in the basins involving these inlets and the ranges of the seas outside during the period of the discharge observations, it will be possible to obtain theoretical values of C according to the methods discussed previously. In table 4 the quantities h_{1m} (i.e. $\sin \tau$) and C are given. Thus it will suffice to take the ratios of the tidal ranges in the basins and the seas and deduce therefrom by referring to table 4 the corresponding values of C. This is shown below for the inlets under consideration.

Inlet	Tidal Range, ft Inside	Tidal Range, ft Outside	h_{1m}	C
Nantucket	2.0	2.5	0.800	0.841
Manasquan	2.5	3.7	0.676	0.825
Beaufort	3.5	4.0	0.878	0.862
Baker's Haulover	3.0	3.3	0.910	0.872

For these four inlets the average theoretical value of C is 0.85, and this compares well with the empirically determined value.

Despite the irregular and rugged shape of the basin areas of the respective basins, the above agreement would indicate that probably in the basins considered the total variations are nearly constant over the entire areas of the basins.

Comparisons with COL Brown's Analysis

The original idea that the flow through an inlet is essentially hydraulic stemmed from COL Brown.[1] The problem was approached by assuming that if the surface fluctuations in the sea are sinusoidal, then the fluctuations of the lagoon also are sinusoidal. Since the resistance of the inlet channel is best described by quadratic laws, the fluctuations in the lagoon fail to conform to a pure sinusoidal variation. The developments on the preceding pages were meant to improve the rigor of analysis. Therefore, it may be worthwhile to make a comparison between the method reported herein and those of COL Brown's analysis for a hypothetical basin. In the review of this report Mr. Wicker made this comparison, and the results are reproduced below from the commentary kindly offered.

Assume: A basin with $A = 400 \times 10^6 \text{ ft}^2$

A channel with $r = 15 \text{ ft}$

$a = 10{,}000 \text{ ft}^2$

$L = 5{,}000 \text{ ft}$

$n = 0.04$

An ocean tide $2H = 4 \text{ ft}$

$H = 2 \text{ ft}$

Then, according to the procedure reported herein, from table 2

$$\frac{AK\sqrt{H}}{a} \times 10^{-4} = 2.040$$

and thus

$$K = 0.36$$

From table 4 for $K = 0.36$ (see also fig. 6)

$$\sin \tau = 0.40 \quad \text{and} \quad C = 0.814$$

Then from equation 62

$$V_m = 3.78 \text{ ft/sec}$$

According to the procedure of COL Brown,

$$Q_t = Ah$$

$$Q_t = 17{,}044a\ V_m = 17{,}044a\ \frac{1.486}{n}\ r^{2/3}\sqrt{\frac{1}{2L}}\ \sqrt[4]{H_s^2 - h^2}$$

where notation is the same as used herein except that

Q_t = total inflow (or outflow)

H_s = range of tide in sea; i.e. $2H$ in the notation of this report

h = range of tide in basin; i.e. $2H_{lm}$ in the notation of this report

Assuming that $h = 1.8$,

$$Q_t = 720 \times 10^6 \text{ ft}^3$$

Also, from the second expression for Q_t in the above,

$$Q_t = 730 \times 10^6 \text{ ft}^3$$

Assuming that $h = 1.9$,

$$Q_t = 760 \times 10^6 \text{ ft}^3$$

$$Q_t = 723 \times 10^6 \text{ ft}^3$$

Assuming that $h = 1.82$,

$$Q_t = 728 \times 10^6 \text{ ft}^3$$

$$Q_t = 727 \times 10^6 \text{ ft}^3$$

Evidently h will be approximately 1.82 ft. Taking this value,

$$V_m = \frac{1.486}{n} r^{2/3} \sqrt{\frac{1}{2L}} \sqrt[4]{H_s^2 - h_1^2}$$

$$V_m = 4.26 \text{ ft/sec}$$

This compares with the 3.78 value obtained by the method herein. COL Brown's procedure overestimates by 12.7 percent as compared with results of the procedure herein.

To take another example, assume:

A basin with $A = 400 \times 10^6 \text{ ft}^2$

A channel with $r = 10$ ft

$a = 5{,}000 \text{ ft}^2$

$L = 10{,}000$ ft

$n = 0.03$

An ocean tide $2H = 6$ ft

$H = 3$ ft

According to the procedure herein, from table 2

$$\frac{AK\sqrt{H}}{a} \times 10^{-4} = 1.519$$

and thus

$$K = 0.1095$$

From table 4, or fig. 6, for $K = 0.1095$,

$$\sin \tau = 0.125 \quad \text{and} \quad C = 0.8107$$

Then from equation 62,

$$V_m = 3.52 \text{ ft/sec}$$

According to the procedure of COL Brown, assuming that $h = 2$,

$$Q_t = 800 \times 10^6 \text{ ft}^3$$

$$Q_t = 329 \times 10^6 \text{ ft}^3$$

Assuming that $h = 1$,

$$Q_t = 400 \times 10^6 \text{ ft}^3$$

$$Q_t = 337 \times 10^6 \text{ ft}^3$$

Assuming that $h = 0.8$,

$$Q_t = 320 \times 10^6 \text{ ft}^3$$

$$Q_t = 338 \times 10^6 \text{ ft}^3$$

Evidently h will be approximately 0.8. Taking this value,

$$V_m = 3.96 \text{ ft/sec}$$

which compares with the value of 3.52 from the procedure reported herein. COL Brown's procedure again overestimates as compared with the results of the method herein. The difference is about the same as previously computed.

Application to Barrier Cuts

The procedure of evaluations relating to inlets, as discussed previously, is equally applicable to barrier cuts separating two bays or the two parts of a bay, provided the coefficient of repletion K is determined anew.

Let Q be the discharge at the cut, V the mean velocity, H_2 the elevation of water surface on one side of the cut, and H_1 the elevation in the protected bay. As before, the elevations are measured with respect to msl. The flow through the cut will be thought to be analogous to flow through an orifice connecting two reservoirs. Then, let C_d be the coefficient of discharge and a the mean cross section of the current at the cut. Hence

$$Q = C_d a \sqrt{2g(H_2 - H_1)} \tag{65}$$

Using the half tidal range H in the outer bay as a measure of surface deflections,

$$V = C_d \sqrt{2gH} \sqrt{h_2 - h_1} \tag{66}$$

Using the surface area of the protected bay A, the storage equation is

$$\frac{dh_1}{d\theta} = \frac{T}{2\pi H} \frac{a}{A} V \tag{11}$$

Eliminating V between equations 11 and 66 yields

$$\frac{dh_1}{d\theta} = K_1 \sqrt{h_2 - h_1} \ , \ h_2 > h_1 \tag{13}$$

$$\frac{dh_1}{d\theta} = -K_1 \sqrt{h_1 - h_2} \ , \ h_1 > h_2 \tag{14}$$

where the coefficient of repletion K_1 now has the new value

$$K_1 = \frac{\sqrt{2gH} \cdot T}{2\pi H} \frac{a}{A} C_d \tag{67}$$

Once K_1 is obtained, the values of C and $\sin \tau$ are read from table 4 or from fig. 6. Using these determinations, the maximum water elevation in the protected part of the bay, the maximum mean current through the cut, and the lag in the oscillations in the protected part are established

from equations 54, 62, and 55, respectively.

Similar to the case of a canal, assuming that the surface oscillations on the two sides of the cut for points sufficiently removed from the cut are nearly sinusoidal, a would be the cross section of the cut when the waters on the two sides of the cut are at msl. As regards the coefficient of discharge C_d, ignoring the effects of the geometry of the cut and the roughness of the passage, this may be put equal to unity.

PART II: APPLICATION TO INDIAN RIVER BAY

Topographic Conditions of Indian River Inlet

At the suggestion of the Committee on Tidal Hydraulics, the case of Indian River Bay, Del., was examined in detail to illustrate the application of the formulas of the present analysis.

Indian River Bay is a lagoon about 2 miles wide from north to south and 6 miles long, the surface area being 15 square miles.[4] The main tributary is the Indian River, which flows into the bay from the west. For about 3-1/2 miles above its mouth the river is broad and relatively shallow, having a depth of about 7 ft at local mean low water (mlw). From this point to the head of navigation, a distance of about 8 miles, it is narrow and sluggish with a meandering course, bordered by mud flats and swamps. Depth of water in the bay averages about 3 ft at local mlw. The bay is connected with the ocean through the Indian River Inlet.

This bay is connected with Rehoboth Bay through two waterways known as "The Ditches," see fig. 8. These passages had a controlling depth of only 2 or 3 ft below the local mlw prior to 1956.

Rehoboth Bay is a shallow-bordered lagoon separated from the ocean by a barrier beach. It is 3 miles wide from east to west and 4 miles long, with a surface area of approximately 13 square miles. The controlling depth is 3 ft below mlw. Two tributary streams, Herring Creek and Love Creek, enter the bay from the west. Lewes and Rehoboth Canal is an inland waterway from Rehoboth Bay to Delaware Bay. From the jettied entrance on the north shore the inland waterway follows a northerly course through marshland, entering fastland in the vicinity of Rehoboth, then in a northwesterly direction it again passes through marshland to an inlet from Delaware Bay, and thence to a connection with Broadkill River. Prior to 1937 the total length was 11.8 miles. The project depth is 6 ft below local mlw and bottom width varies from 50 to 100 ft. It must be kept in mind that the connection of Lewes and Rehoboth Canal with Delaware Bay via Roosevelt Inlet was not in existence in 1928-1929. The length given here is for a connection with Delaware Bay via Broadkill River.

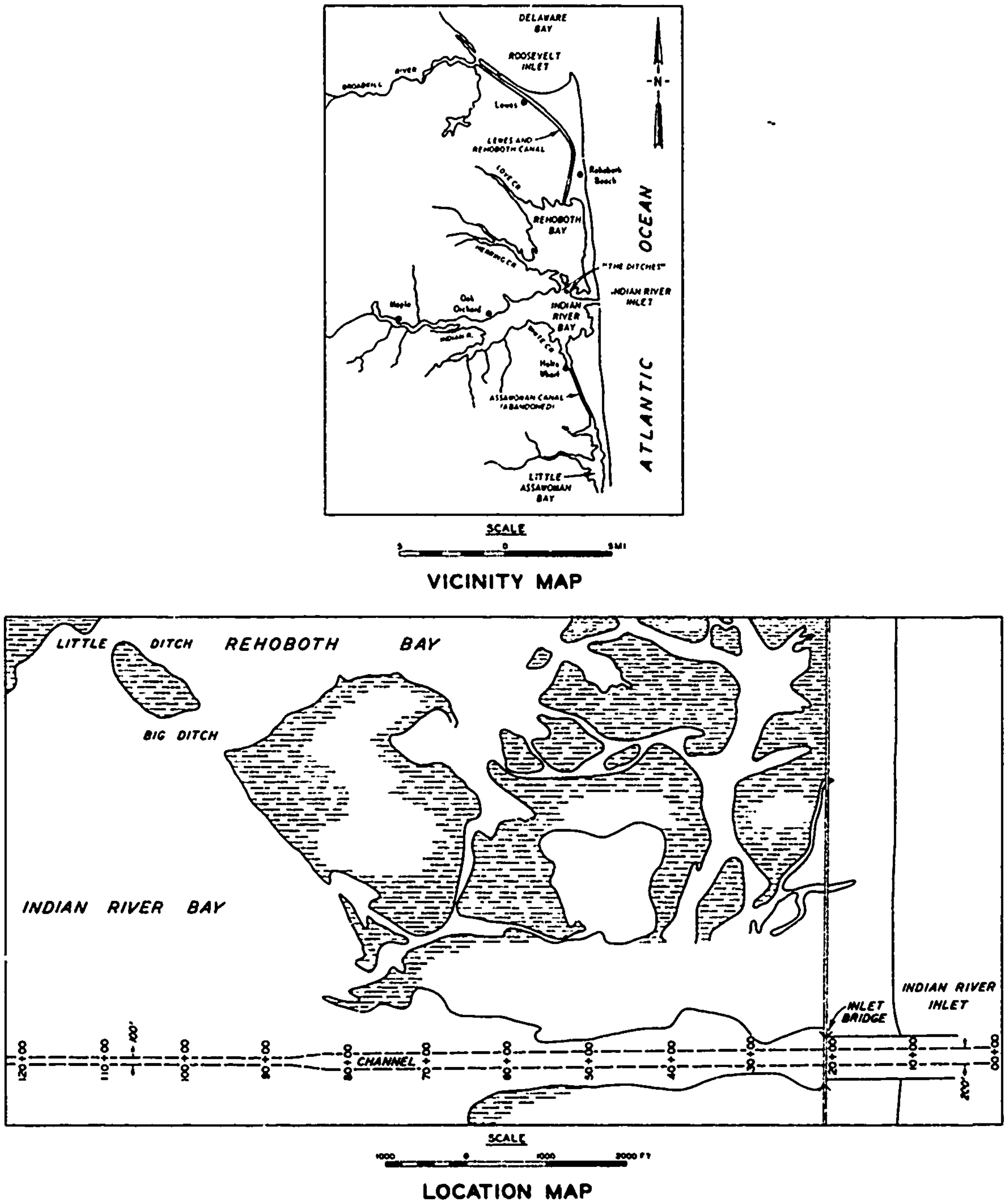

Fig. 8. Location and vicinity maps of Indian River Bay and Rehoboth Bay

Indian River Inlet

Indian River Inlet has an interesting history.[5] Prior to the modern improvements it was an unstable passage, opening and closing alternately. Under such conditions an inlet is likely to move from one place to another, and evidence suggests that during the period from 1847 to 1910 this inlet migrated northward a distance of 2 miles. About 1910 the natural inlet had shoaled to an extent that a closing was impending. In fact, about the year 1925 the closing was complete and little hope could be entertained for the opening under natural causes. In the summer of 1928 the Indian River Commission undertook to open the inlet by making a cut through the barrier reef at a point almost opposite the center of Indian River Bay and 1 mile south of the final position of the natural inlet. A channel 60 ft wide and 4 ft deep at mlw of the ocean was excavated from the bay, beginning where the water was 4 ft deep on the same datum, extending eastward through the marsh, and then partly through the barrier reef to within 115 ft of the high-water mark on the ocean beach. Actual work began on 23 June 1928 and was finished in October. From the eastern end of the dredged cut an attempt was made to complete the opening to the ocean by first removing a part of the sand with scrapers and then blasting out a channel. The final attempt in November 1928 was not successful, and as the funds were exhausted, work was suspended. About a month after the work had stopped the locality regained its former appearance except for the opening through the dunes and the cut in the marsh. The trench made by the explosions was obliterated. The ocean end had been filled to above low water for a distance of 125 ft. At the end of April 1929, locally interested people made a small cut, and the impounded water eroded an opening through the remainder of the barrier beach. The cross section of this opening was estimated to be 100 ft wide and 4-1/2 ft deep in June 1929, but late that summer it was evident that the inlet was shoaling rapidly and again was about to close entirely. In November 1929, dredging of a channel 60 ft wide at the bottom and 8 ft deep at mlw was commenced, beginning in the marsh about 3700 ft from the ocean. The excavation followed the line of the previous cuts, and by January 1930 it

had reached the outer bar. The work provided an inlet that was considered to be comparable in size to the old inlet that existed prior to 1910. However, by November 1930 the inner bar was above mlw and the controlling depth over the outer bar was 2 ft. By 1935, the inlet was virtually closed and the State of Delaware decided again to appeal to the Federal Government for assistance in providing a stabilized inlet.

As expressed by the state and local interests, the purposes of a stable inlet were as follows: to provide for an adequate exchange of water between the bays and the ocean in order to relieve stagnation and to increase the salinity of the bays sufficiently to support a seafood industry, and to provide a satisfactory channel for navigation from the ocean to points within the bays. The project designed by the U. S. Army Corps of Engineers for these indicated purposes consisted of jetties at the inlet entrance extending to the 14-ft depth contour in the ocean, a channel 15 ft deep and 200 ft wide from the end of the jetties to a point 7000 ft inside Indian River Bay, and thence a channel 6 ft deep and 100 ft wide to the natural 6-ft depth contour in Indian River Bay. This project was approved by Congress, and construction was completed in 1939. A generally satisfactory inlet has existed since that time; as of 1966, a total of approximately 100,000 cu yd of maintenance dredging had been performed and the controlling depths in the inlet have approximated 9 ft, which is adequate for navigation. The quality of the water within the bays apparently has been satisfactory since opening of the inlet.

Profile of Tide in Indian River Bay

One of the assumptions used in the derivation of the formulas relating to the regime of tides in an internal basin was that the mean levels of the water in the basin and of the sea outside are contained in the same horizontal plane. It is necessary to examine how nearly this assumption is satisfied for Indian River Bay. In table 5, which was prepared by Mr. Wicker for the guidance of the discussions herein, the longtime tide data of the various stations of the two bays are given. The observations of the surveys of the two periods, 1938-1939 and 1948 and 1950, appear to

be distinct. The former covers a period of 14 months. From the 1938-1939 survey a superelevation of mean tide level (mtl) of about 0.15 ft is indicated. The correspon.ing value in Rehoboth Bay, taking the average from Dewey Beach and Love Creek, is 0.08 ft. With reference to the 1948 and 1950 survey covering a period of 6-1/2 months, a much augmented superelevation of mtl at Oak Orchard and Dewey Beach over that observed in 1938-1939 is indicated.

The averaged 1948-1950 values are shown in fig. 9. These were

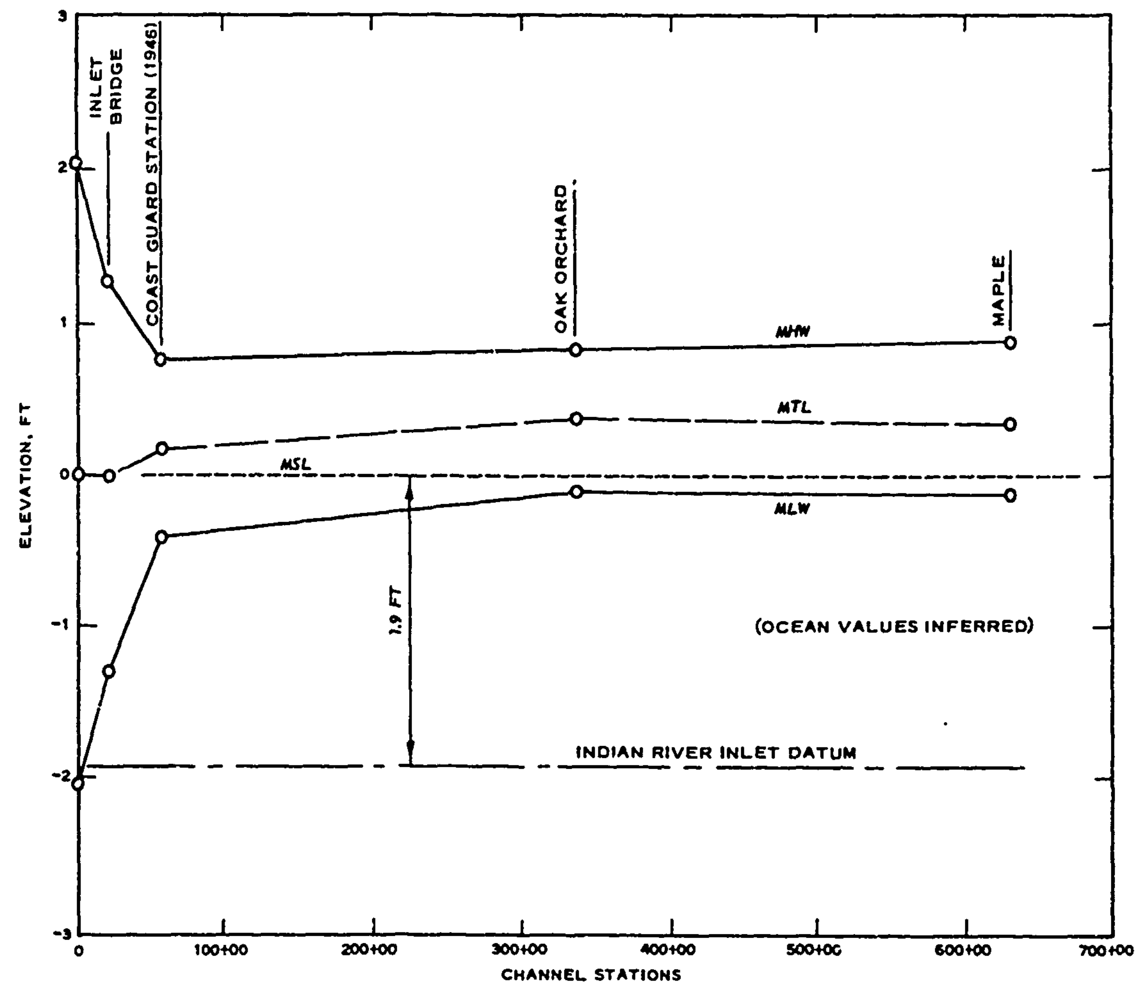

Fig. 9. Indian River Inlet and Bay 1948-1950 tides reduced to longtime means

selected for detailed study in lieu of 1938-1939 values because current velocity observations were made in 1948 in Indian River Inlet and at stations in the bay. Ocean values were not observed. Some measurements would place the range of tide at the ocean as 3.77 ft. However, in a memorandum dated 12 December 1939, Mr. Wicker rejected this value on the ground that the observations were very poor and very few in number. On the premise that the tide at Indian River Inlet is most likely similar to the tide at Lewes, the value 4.1 ft was proposed for the range. The value of tide at Lewes is well established. This value is shown in fig. 9, $H = 2.05$, and will be used in all the subsequent computations.

Fig. 9 also shows that the range of tide along the inlet decreases almost uniformly. In the bay area, tidal displacements are nearly constant. The mtl at the Inlet Bridge coincides with msl, whereas in the bay interior a superelevation of 0.3 ft is established. This is a superelevation much higher than that observed during the surveys of 1938-1939. The tributary discharges, the shallowness of the inlet, and the wind effects must have a bearing on the superelevations. It is desirable, therefore, that each of these factors be separately examined in order to establish a reasonable limiting value for the superelevation.

Tributary Discharges into the Bays

In 1930 the Indian River Inlet Commission (IRIC) conducted a field investigation to ascertain the inflows from the sea and the outflows during a tidal cycle, and also the changes in the water level within the bays. The level changes were noted for a period of about two years at Holts Wharf, and these are shown in fig. 10 together with the monthly precipitation in the area. The flows were measured by means of current meters at the Rehoboth Bay end of Lewes and Rehoboth Canal and at Indian River Inlet. There was no need for flow observation in Assawoman Canal as this waterway was closed during this study. Observations of currents were made continuously each day except Sunday for a period of three months. Average values from these observations for consecutive periods are shown in table 6 together with the average durations of inflow and outflow.

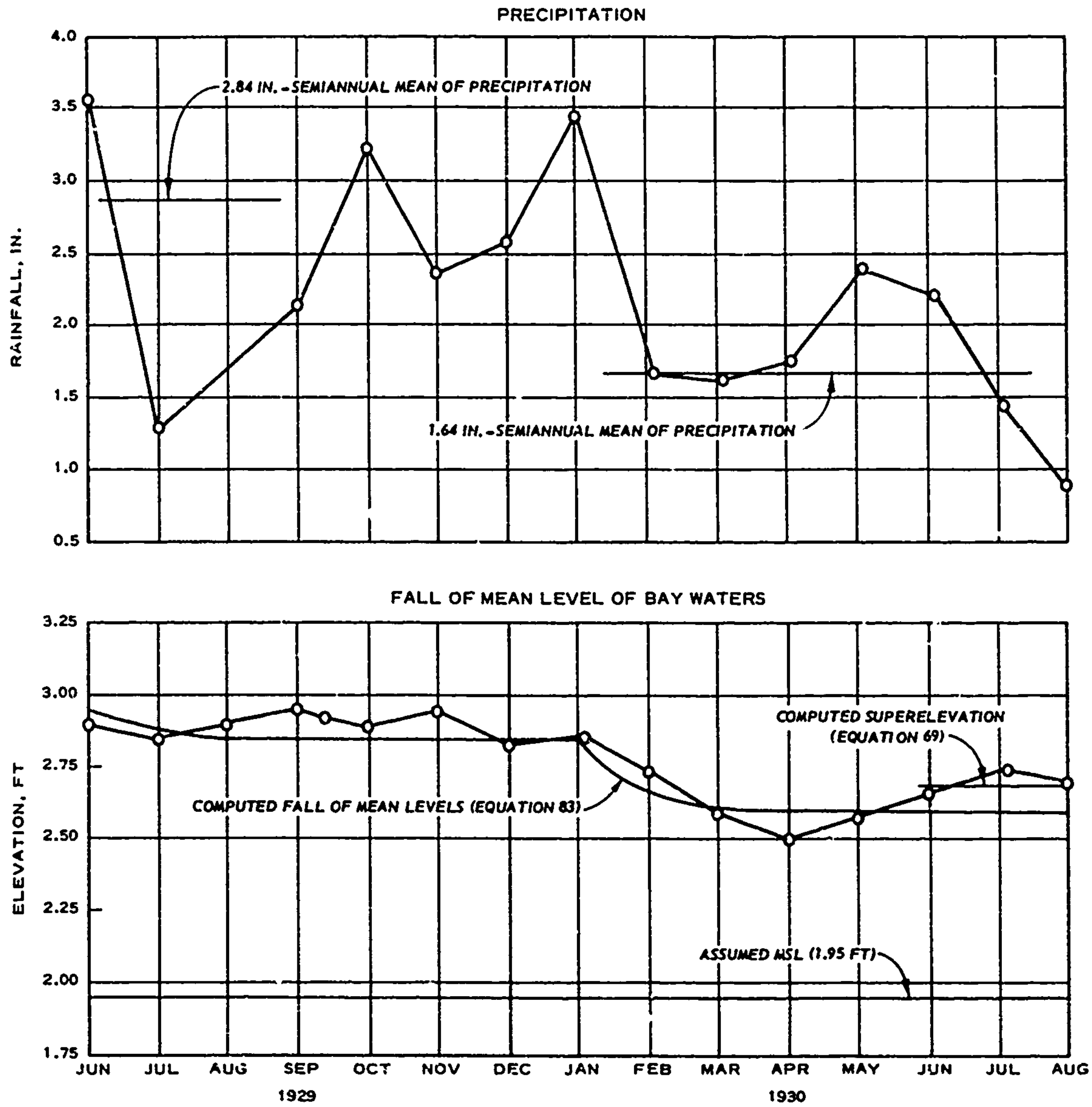

Fig. 10. Bay water elevation and precipitation
(Survey of Indian River Inlet Commission)

For inlet and canal alike, the outflow is greater than the inflow. This is attributed to the fact that at the time the average level of the water of the bays was above that of the sea. The value of the superelevation can be determined analytically, since the disparity in the inflow and the outflow durations is known. Let Δ be the superelevation, H the semirange of the tide in the sea, and H_2 the sea tidal elevation

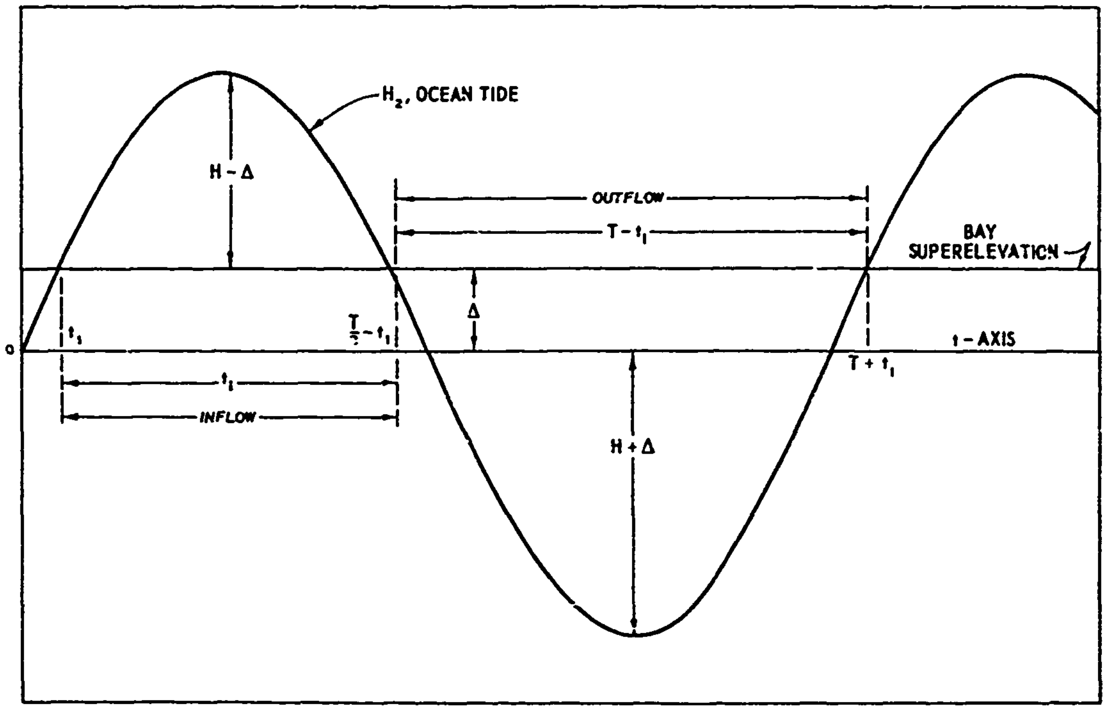

Fig. 11. Notation diagram. Ocean tide and bay superelevation

measured from msl, approximately $H = 2.05$ ft (see fig. 11). The tide of sea is

$$H_2 = H \sin \sigma t \ , \ \sigma = 2\pi/T$$

if t is measured from the instant when the tide is crossing the msl plane and is increasing in value. The period of tide is T. Let t_1 be the first instant when the waters of the bays and of the sea are in the same horizontal plane. Let t_i be the duration of the inflow. Then $t_1 + t_i = \frac{T}{2} - t_1$ is the next instant when the waters of the bays and of the sea are again in the same horizontal plane. Since the internal tides of the bays at the time of the flow measurements were negligible, it can be written that

$$\Delta = H \sin \sigma t_1$$

and also that

$$\Delta = H \sin (\sigma t_1 + \sigma t_i)$$

Eliminating Δ between these two equations, expanding the result, and collecting terms, it can be seen finally that

$$\tan \sigma t_1 = \frac{\sin \sigma t_i}{1 - \cos \sigma t_i}$$

Since σt_1 is a small angle

$$\tan \sigma t_1 = \sigma t_1 = \Delta/H \tag{68}$$

and

$$\Delta/H = \frac{\sin \sigma t_i}{1 - \cos \sigma t_i} \tag{69}$$

From table 6 it is seen that the average inflow duration is 4.46 hr, whereas the average outflow duration is 7.14 hr. Since the sum of these two durations, 11.60 hr, is less than 12.42 hr, the period of the tides, the observed inflow and outflow duration values need to be increased 1.072 times. Thus the adjusted value of the inflow duration is 4.79 hr and then σt_i = 139.2 degrees. Evaluating the right side of equation 69 yields

$$\Delta/H = 0.372$$

and

$$\Delta = 0.372 \times 2.05 = 0.76 \text{ ft}$$

Accordingly, if it is assumed that during the study the msl was 1.95 ft above the 1929 datum, the mean level elevation in the two bays should be 2.71 ft. This is shown also in fig. 10, [illegible] it appears that the value is in close agreement with the Holts Wharf measurement of the time. In the

subsequent analysis the notation Δ_o will stand for 0.76 ft superelevation.

Examination of the lower plot in fig. 10 shows that the level of the bay waters fell about 1/2 ft from June 1929 to April 1930. It might be helpful to obtain a theoretical relation to show the dependence of the rate of fall upon the tributary discharges.

First an expression is derived relating the net total outflow Q_T during a tide, for a superelevation Δ, assuming in the analysis that the variations of canal and inlet cross sections during ebb and flood are ignorable, and, more important, that the tidal changes inside are insignificant, $H_1 = \Delta$. According to equation 6 the inflow during a tidal cycle is given by, after putting $m = 1$ and $H_1 = \Delta$,

$$Q_i = ka \int_{t_1}^{\frac{T}{2}-t_1} \sqrt{\frac{H_2}{H} - \frac{\Delta}{H}}\, dt \tag{70}$$

and the outflow by

$$Q_o = ka \int_{\frac{T}{2}-t_1}^{T+t_1} \sqrt{\frac{H_2}{H} + \frac{\Delta}{H}}\, dt \tag{71}$$

where

$$k = \sqrt{\frac{2grH}{\lambda L + r}}$$

$$H_2 = H \sin \sigma t \ , \ \sigma = 2\pi/T$$

$$\sigma t_1 = \Delta/H$$

and a is the cross-sectional area of canal or inlet. Inspecting fig. 11 it will be inferred that as a good approximation

$$\frac{H_2}{H} - \frac{\Delta}{H} = \left(1 - \frac{\Delta}{H}\right) \sin \frac{2\pi\theta}{T - 4t_1} \left.\begin{matrix} \theta = t - t_1 \\ 0 \leq \theta \leq \frac{T}{2} - 2t_1 \end{matrix}\right\}$$

and

$$\frac{H_2}{H} + \frac{\Delta}{H} = \left(1 + \frac{\Delta}{H}\right) \sin \frac{2\pi\theta}{T + 4t_1} \left\} \begin{array}{l} \theta = t - \frac{T}{2} + t_1 \\ \frac{T}{2} - t_1 \leq \theta \leq \frac{T}{2} + 2t_1 \end{array}\right.$$

Using the relation in equation 30 and ignoring the higher harmonics

$$\sqrt{\frac{H_2}{H} - \frac{\Delta}{H}} = N_1 \left(1 - \frac{\Delta}{H}\right)^{1/2} \sin \frac{2\pi\theta}{T - 4t_1}$$

and

$$\sqrt{\frac{H_2}{H} + \frac{\Delta}{H}} = N_1 \left(1 + \frac{\Delta}{H}\right)^{1/2} \sin \frac{2\pi\theta}{T + 4t_1}$$

Since $\frac{\Delta}{H}$ is a small fraction, sufficient approximation is obtained by using

$$\sqrt{\frac{H_2}{H} - \frac{\Delta}{H}} = N_1 \left(1 - \frac{1}{2}\frac{\Delta}{H}\right) \sin \frac{2\pi\theta}{T - 4t_1} \tag{72}$$

and

$$\sqrt{\frac{H_2}{H} + \frac{\Delta}{H}} = N_1 \left(1 + \frac{1}{2}\frac{\Delta}{H}\right) \sin \frac{2\pi\theta}{T + 4t_1} \tag{73}$$

Substituting in equations 70 and 71 from equations 72 and 73, respectively, and carrying out the required integration, yields

$$Q_i = \frac{N_1}{\pi} \left(1 - \frac{1}{2}\frac{\Delta}{H}\right) \left(1 - \frac{4t_1}{T}\right) Tka$$

and

$$Q_o = \frac{N_1}{\pi} \left(1 + \frac{1}{2}\frac{\Delta}{H}\right) \left(1 + \frac{4t_1}{T}\right) Tka$$

Since

$$\frac{4t_1}{T} = \frac{2\sigma t_1}{\pi} = \frac{2}{\pi}\frac{\Delta}{H}$$

and since the square of Δ/H is negligible with respect to unity, the expressions of inflow and outflow simplify to

$$Q_i = \frac{N_1}{\pi}\left[1 - \left(\frac{1}{2} + \frac{2}{\pi}\right)\frac{\Delta}{H}\right] Tka \tag{74}$$

and

$$Q_o = \frac{N_1}{\pi}\left[1 + \left(\frac{1}{2} + \frac{2}{\pi}\right)\frac{\Delta}{H}\right] Tka \tag{75}$$

For Δ positive, the inflow is smaller than the outflow. That is,

$$Q_T = Q_o - Q_i \tag{76}$$

Introducing from equations 74 and 75, the net outflow becomes

$$Q_T = \mu a \; T\Delta/H \tag{77}$$

where

$$\mu = \frac{N_1}{\pi}\left(1 + \frac{4}{\pi}\right) k \tag{78}$$

The product μa is referred to as the conveyance of an inlet or canal. It is not a dimensionless quantity; its dimensions are $(\mu a) = (L^3T^{-1})$, i.e. volume per unit time. In the early periods when the Indian River was being developed, the dimensions and hydraulic characteristics of the inlet were somewhat uncertain; therefore, for the analysis of the tidal regime of that time, the conveyances could not be evaluated and an alternate approach was necessary. During the period from June 1930 to September 1930, the net total outflow of the inlet and canal was recorded. Let Q_{T_o} be this outflow and Δ_o represent the prevailing superelevation of the bay waters at the time. From table 6, the combined value of the net outflow is $Q_{T_o} = 12.76 \times 10^6$ ft^3. Analysis places Δ_o at 0.76 ft. Let Q_T be the total net outflow at some other superelevation Δ. Then

$$Q_{T_o} = \mu a \; T \; \Delta_o/H$$

and

$$Q_T = \mu a \; T \; \Delta/H$$

(It is assumed that the longtime ocean range of tide is invariable.) Taking the ratio, it follows that

$$Q_T = Q_{T_o} \; \Delta/\Delta_o \tag{79}$$

showing that net outflow is proportional to superelevation.

The longtime storage equation for the two bays is, ignoring internal tides,

$$A \frac{d\Delta}{dt} = \frac{Q_r}{T} - \frac{Q_T}{T}$$

where Q_r is the river inflow for one tidal cycle. According to weather bureau data, the normal annual average evaporation, as judged from the evaporation in Class A pans, is about 39 in., whereas the normal annual precipitation is 40 in. in northern Delaware. Since these two values are nearly equal, the matter of precipitation and evaporation does not enter into the storage equation. Introducing from equation 79,

$$A \frac{d\Delta}{dt} = \frac{Q_r}{T} - \frac{Q_{T_o}}{T} \frac{\Delta}{\Delta_o} \tag{80}$$

To place the equation in a dimensionless form, first introduce T_1, the number of seconds in a year, and Δ_1, the initial elevation of the bay water at time $t = 0$. Adopting the new variables,

$$\left.\begin{aligned} \delta &= \Delta/\Delta_1 \\ \text{and} \qquad \tau_1 &= t/T_1 \end{aligned}\right\} \tag{81}$$

and introducing them in equation 80, the latter changes to

$$\frac{d\delta}{d\tau_1} = \alpha_1 - \beta_1\delta \tag{82}$$

where

$$\alpha_1 = \frac{Q_r}{A\Delta_1} \cdot \frac{T_1}{T}$$

and

$$\beta_1 = \frac{Q_{T_o}}{A\Delta_1} \cdot \frac{T_1}{T} \cdot \frac{\Delta_1}{\Delta_o}$$

The solution for δ , subject to the condition that $\delta = 1$ when $\tau_1 = 0$, is

$$\delta = \frac{1}{\beta_1}\left[\alpha_1 - \left(\alpha_1 - \beta_1\right) e^{-\beta_1\tau_1}\right] \tag{83}$$

This equation can be used to determine the discharges from the rivers if the fall of the surface waters in the bays is known and, conversely, the fall of the water surface if the river discharges are known.

According to page 3 of the IRIC report, the freshwater flow into the bays amounts to 30×10^6 ft^3 per day. This is 15×10^6 during 12 hr, which amounts to 348 ft^3/sec. The drainage area of the bays, including their water-surface areas, is 254.5 square miles. Deducting the water-surface areas, amounting to 29.5 square miles, assuming that evaporation is nearly of the same value as the precipitation, the net drainage area equals 225 square miles. Thus, the unit runoff is 1.55 ft^3 per second per square mile. The only area currently gaged is a tributary of Indian River with a drainage area of 5.24 square miles. The average discharge through 1963 is 741 ft^3/sec, and the unit runoff is 1.41 ft^3 per second per square mile. The total drainage area of 225 square miles does not differ materially as to runoff characteristics from any part thereof. It appears that the IRIC value of 30×10^6 ft^3 per day or 15×10^6 for 12 hr is a reasonably

accurate value and will be used in the subsequent computations.

It would be instructive to find how closely the fall of the water level during the period from June 1929 to August 1930 could be determined using equation 83. It will be assumed that the discharges from the tributaries were in proportion to the precipitation of the area, and that during this period the conveyances of Indian River Inlet and Lewes and Rehoboth Canal remained constant. In particular, on the yearly average basis, $Q_r = 15 \times 10^6$ ft^3 per tidal cycle and $Q_{T_o} = 12.76 \times 10^6$ ft^3 per tidal cycle when the superelevation is $\Delta_o = 0.76$ ft. Examining the upper graph in fig. 10 it is seen that the average monthly precipitation from June 1929 to January 1930 was 2.84 in., whereas from January 1930 to August 1930 it was 1.64 in. It is now inferred that for the former period $Q_r = 19.0 \times 10^6$ ft^3, and for the later period $Q_r = 11.0 \times 10^6$ ft^3. Accordingly, the computation of the fall of water surface in the bays should be made separately for each period.

For the first period it is noted that the water-surface elevation in June 1929 was about 2.95 ft. This makes $\Delta_1 = 1$ ft. The relevant data for the fall computation are:

$$\Delta_1 = 1 \text{ ft}, \qquad \Delta_o = 0.76 \text{ ft}$$
$$A = 8.2 \times 10^8 \text{ ft}^2, \quad T/T_1 = 730$$
$$Q_r = 19 \times 10^6 \text{ ft}^3, \qquad Q_{T_o} = 12.8 \times 10^6 \text{ ft}^3$$

Accordingly, $\alpha_1 = 13.8$ and $\beta_1 = 15.1$, and with these values equation 83 becomes

$$\delta = 0.91 + 0.09\, e^{-15.1\tau}$$

and as $\Delta_1 = 1$ ft

$$\Delta = 0.91 + 0.09\, e^{-15.1\tau}$$

The fall curve corresponding to this is shown in fig. 10. In January 1930

the water-surface elevation is 2.86 ft and this makes $\Delta_1 = 0.91$ ft. The relevant data for the fall computation in the second period are:

$$\Delta_1 = 0.91 \text{ ft} , \qquad \Delta_o = 0.76 \text{ ft}$$
$$A = 8.2 \times 10^8 \text{ ft}^2, \; T/T_1 = 730$$
$$Q_r = 11 \times 10^6 \text{ ft}^3 , \quad Q_{T_o} = 12.8 \times 10^6 \text{ ft}^3$$

Accordingly, $\alpha_1 = 10.7$ and $\beta_1 = 15.1$, and with these values equation 83 becomes

$$\delta = 0.71 + 29 \, e^{-15.1\tau}$$

and as $\Delta_1 = 0.91$ ft

$$\Delta = 0.65 + 0.26 \, e^{-15.1\tau}$$

The fall curve corresponding to this is also shown in fig. 10. The agreement between the observed and computed falls is sufficiently close, suggesting that the use of equation 83 is permissible to obtain the limiting mean elevation of bay waters for the later periods with the Indian River Inlet cross section greatly augmented.

Limiting Mean Elevation of Bay Waters

With the later enlargement of Indian River Inlet the conveyance was increased and the mean level of the bay waters descended to a lower plane. It will now be examined as to what the limit would be for the inlet condition present during the surveys of 1938-1939. First, however, it is necessary to establish the conveyance of the enlarged channel.

According to the IRIC report, the inlet cross section during the study varied from 440 to 750 ft^2. Similar changes also occurred in the depth of channel. For example, the cut through the barrier reef had

shoaled from -8.0 ft (zero at 1929 mlw) to which it was dredged to an average of about -2 ft, indicating a general fall of 6 ft. This would mean that the depth varied from 4 to 10 ft. Taking the mean of the extremes, the lower value repeated, the channel cross section would be 543 ft^2 and the depth 6 ft. It is seen, next, from table 7 that in 1938-1939 the average cross section of the inlet channel in the part next to the sea, east of sta 22+00, was 6240 ft^2, whereas the depth was 11.15 ft. Since the conveyance of the channel would be proportional to

$$r^{2/3}a$$

then the conveyance of the inlet channel in 1938-1939 was about 17.6 times as large as the conveyance of the channel of 1930. During the period of the IRIC study, the net outflow per tidal cycle through the inlet channel as discussed previously was 6.38×10^6 ft^3, corresponding to a superelevation of bay waters of 0.76 ft from msl. The corresponding outflow capability through the 1938-1939 inlet channel would be 17.6 times as large, i.e.,

$$17.6 \times 6.38 \times 10^6 = 112.2 \times 10^6 \text{ ft}^3$$

Meanwhile, since no significant changes had occurred in the Lewes and Rehoboth Canal, the net outflow per tidal cycle through the canal remained the same and thus it is

$$6.38 \times 10^6 \text{ ft}^3$$

Then, adding these two, the net outflow capability per tidal cycle in 1938-1939 through the two waterways was

$$Q_{T_o} = 118.6 \times 10^6 \text{ ft}^3, \Delta_o = 0.76$$

The tributary discharges also remained the same:

$$Q_r = 15 \times 10^6 \text{ ft}^3$$

To determine the superelevation of the bay waters, assume that at $t = 0$, $\Delta_1 = \Delta_o = 0.76$. Also:

$$A = 8.2 \times 10^8 \text{ ft}^2$$

$$Q_r = 15 \times 10^6 \text{ ft}^3 \text{ per tidal cycle}$$

$$Q_{T_o} = 118.6 \times 10^6 \text{ ft}^3 \text{ per tidal cycle}$$

$$T/T_1 = 730$$

Hence, $\alpha_1 = 17.5$ and $\beta_1 = 138.2$, and equation 89 reduces to

$$\delta = 0.127 + 0.873 \; e^{-138\tau}$$

and since $\Delta_1 = 0.76$ ft, also

$$\Delta = 0.097 + 0.664 \; e^{-138\tau}$$

Accordingly, the terminal superelevation which is reached in about 30 days has the value 0.097 ft. This agrees fairly well with the longtime observed superelevation of the 1938-1939 surveys at Oak Orchard and Love Creek (see table 5).

Effect of Inlet Cross Section on Bay Water Levels

In inlets where the mean depth of water is small, the cross section during a flood would be greater than that during an ebb. Assuming that there are no tributaries leading to a basin and that the water level of the bay has attained its equilibrium position, the inflow and outflow volumes for a tidal cycle would be the same. Since the currents of a flood would be greater than those of an ebb, the duration of flood would be smaller than the duration of ebb. This circumstance leads to a

superelevation Δ which may be related to the half tidal range of the sea H. It should be sufficient to examine this relation for the simple case of insignificant internal tide.

Because surface width is very large in comparison with depth, the changes in the cross section would come from the changes in the depth, r. Thus, for the instantaneous discharge,

$$\Delta q = M_1 \, r^{5/3} \sqrt{\Delta H} \tag{84}$$

where ΔH is the differential head between the bay and the ocean water and M_1 a factor of proportionality involving inlet length, roughness, and surface width. The depth of water at mid-inlet can be taken to give the effective hydraulic radius, and on the assumption that the surface water in the inlet is nearly straight, its value accordingly would be

$$r = r_o + \frac{\Delta}{2} + \frac{H}{2} \sin \theta \, , \; \theta = \sigma t \tag{85}$$

if the ocean tide is given by

$$H_2 = H \sin \sigma t$$

Here r_o is the water depth at time $t = 0$, t being measured from the instant the tide is rising from the msl plane. Equation 85 can be written in the form

$$r = r_o \left(1 + \frac{\Delta}{2r_o} + \frac{H}{2r_o} \sin \theta \right)$$

and since $\Delta/2r_o$ is small, also

$$r = r_o \left(1 + \frac{\Delta}{2r_o} \right) \left(1 + \frac{H}{2r_o} \sin \theta \right) \tag{86}$$

Denoting by Δq_1 the instantaneous discharge during an ebb and by Δq_o the instantaneous discharge during a flood, substituting in equation 84 from equation 86, and recalling the meaning of H:

$$\Delta q_i = M_2 \left(1 + \frac{H}{2r_o} \sin \theta\right) \sqrt{\sin \theta - \frac{\Delta}{H}} \qquad (87)$$

and

$$\Delta q_o = M_2 \left(1 + \frac{H}{2r_o} \sin \theta\right) \sqrt{\frac{\Delta}{H} - \sin \theta} \qquad (88)$$

where

$$M_2 = M_1 \sqrt{H}\, r_o \left(1 + \frac{\Delta}{2r_o}\right)$$

and this is a constant.

At the instants t_1 , $\frac{T}{2} - t_1$, and $T + t_1$, the waters of the bays and of the ocean are at the same level. Denote σt_1 by θ_1 .

$$\Delta = H \sin \theta_1$$

and since θ_1 is a small angle

$$\theta_1 = \Delta/H \qquad (89)$$

Denote total inflow during an ebb by Q_i and total outflow by Q_o . From equations 87 and 88:

$$Q_i = M_2 \int_{\theta_1}^{\pi-\theta_1} \left(1 + \frac{H}{2r_o} \sin \theta\right) \sqrt{\sin \theta - \frac{\Delta}{H}}\, d\theta$$

and

$$Q_o = M_2 \int_{\pi-\theta_1}^{2\pi+\theta} \left(1 + \frac{H}{2r_o} \sin \theta\right) \sqrt{\frac{\Delta}{H} - \sin \theta}\, d\theta$$

In the absence of the tributaries $Q_i = Q_o$ and hence

$$\int_{\theta_1}^{\pi-\theta_1} \left(1 + \frac{H}{2r_o} \sin \theta\right) \sqrt{\sin \theta - \frac{\Delta}{H}}\, d\theta$$

$$= \int_{\pi-\theta_1}^{2\pi+\theta_1} \left(1 + \frac{H}{2r_o} \sin \theta\right) \sqrt{\frac{\Delta}{H} - \sin \theta}\, d\theta \qquad (90)$$

which defines implicitly the relation between Δ/H and $H/2r_o$. A simple relation can be derived if it is assumed that both Δ/H and $H/2r_o$ are small quantities, so small that the products or the squares of these ratios are negligible in comparison with unity. This will be granted. Then quite adequately one can put

$$\sqrt{\sin \theta - \frac{\Delta}{H}} = \left(1 + \frac{1}{2}\frac{\Delta}{H}\right) \sin \theta - \frac{\Delta}{H}, \quad \theta_1 \leq \theta \leq \pi - \theta_1$$

$$\sqrt{\frac{\Delta}{H} - \sin \theta} = \frac{\Delta}{H} - \left(1 - \frac{1}{2}\frac{\Delta}{H}\right) \sin \theta, \quad \pi - \theta_1 \leq \theta \leq 2\pi + \theta_1$$

Introducing these in equation 90 and carrying out the integrations and neglecting in the result the second-order quantities, one has

$$\frac{\Delta}{H} = \frac{1}{4}\frac{H}{r_o} \qquad (91)$$

Applying the result to the case of the Indian River Inlet with $H =$ 2.05 ft and $r_o = 12$ ft one finds that

$$\Delta = 0.088 \text{ ft}$$

The deduction to be made from the discussions of the last two sections is that although there are river discharges and the inlet waterway is moderately shallow, nevertheless, the level of the waters of the two bays is expected to be only slightly elevated. According to the estimates made, the superelevation from msl could hardly be greater than 0.20 ft,

a value considerably less than that shown in fig. 9, but close to that of 1938-1939, and could very well be ignored in the computations to be entered subsequently.

Wind Tides in Indian River Bay

Examining again in fig. 10 the course of the elevation of the bay waters, it is seen that there was a fall from September 1929 to April 1930 and then a continued rise up to about 2.80 ft. This rise may be explained in part by the shoaling of the inlet. The consequent decreases in the outflow amounts could account for the impounding of the waters of the bays.

Besides this, setup produced by the winds would be another factor to bear in mind. The frequently occurring strong winds can have a measurable bearing on the exchange of waters between the two bays and the ocean. A strong wind blowing westward would lower the water surface at the interior ends of inlet and canal and at the same time increase the height of the ocean waters at the exterior ends of the two passages. The additional head thus created may augment the inflow to a degree that a net inflow is realized during a complete tidal cycle.

Wind setup is governed by the equation

$$\left.\begin{aligned} \frac{d\Delta}{dx} &= -\frac{\tau_s}{\rho g\,(D_o + \Delta)} \\ \tau_s &= \chi\,\rho_a\,V^2 \end{aligned}\right\} \qquad (92)$$

Where Δ is the setup, i.e. the displacement of the surface of water from the undisturbed level; D_o is the depth of the undisturbed water; τ_s is the wind stress; ρ and ρ_a are the densities of water and air, respectively; V is the wind velocity; and χ is the Taylor stress coefficient. The analysis of setup in Lake Erie due to severe storms suggests that $\chi \cong 0.0025$ (reference 6). Let Δ_w denote the lowering of waters, measured from the undisturbed level, at the inlet interior end. If L is the length of the bays, the solution of equation 92 subject to the condition that Δ is small in comparison with D_o, is

$$\Delta_w = \frac{1}{2} \chi \rho_a V^2 L/g D_o \rho \tag{93}$$

Taking the average of the lengths of the two bays as the operative basin lengths, L = 4.5 miles (approximately). Putting $\rho/\rho_a = 840$, one finds from equation 93 that $\Delta_w = 0.045, 0.18, 0.40, 0.72,$ and 1.10 ft corresponding to wind velocities V = 10, 20, 30, 40, and 50 mph, respectively.

Next, one should consider the rise of water surface at the ocean end of the inlet or canal. Assume that the Continental Shelf is of length L and that the depth of water at the point where the shelf terminates is of value D_o. Assume that the shelf is a uniformly inclining surface of inclination α_2 so that $D_o = \alpha_2 L$. Since the depths are great beyond the shelf, it would be appropriate to suppose that wind setup vanishes at the ocean end of the shelf. Measure x from the shoreline, and denote the wind tide at x_o as Δ_o.

Writing

$$\left.\begin{aligned} \kappa &= \frac{\tau_s L}{\rho g D_o^2} \;, \quad \tau_s = \chi \rho_a V^2 \\ \eta &= \frac{\Delta}{D_o} \;, \quad \eta_o = \frac{\Delta_o}{D_o} \\ \zeta &= x/L \end{aligned}\right\} \tag{94}$$

and

the wind tide equation, equation 92, takes on the form

$$\frac{d\eta}{d\zeta} = -\frac{\kappa}{\zeta + \eta} \tag{95}$$

with the boundary conditions

$$\eta = \eta_o \;, \quad \zeta = 0$$

$$\eta = 0 \;, \quad \zeta = 1$$

For the problem at hand an approximate solution of equation 95 should suffice. As the water-surface height should not differ substantially

from a straight line disposition,

$$\eta = \eta_o(1 - \zeta)$$

When substituted in the right-hand member of equation 95, the result of this is

$$\frac{d\eta}{d\zeta} = - \frac{\kappa}{\eta_o + (1 - \eta_o)\ \zeta}$$

The solution of the latter, subject to the condition that $\eta = 0$ at $\zeta = 1$, is

$$\eta = - \frac{\kappa}{1 - \eta_o} \log\ [\eta_o + (1 - \eta_o)\ \zeta]$$

and as $\eta = \eta_o$ when $\zeta = 0$, by substitution

$$\frac{\kappa}{1 - \eta_o} \log \frac{1}{\eta_o} = \eta_o \tag{96}$$

an expression that relates, implicitly, the rise of ocean water at the coastline with the velocity of the wind.

Assume that the Continental Shelf seaward of the inlet is 25 miles long and that the slope is $\alpha = 1/100$. According to equation 96 the wind which should raise the inlet ocean waters by 1/10 ft is of the strength of 62 mph.

An additional rise of water level would be due to the drift current from the wind. The wind velocity being V, the drift current (i.e. the velocity of water at the surface) would amount to $0.03\ V$. Besides the rise of the water surface at the coastline, the wind will cause a drag on the waters of the inlet, augmenting the flow that would result from a difference of water levels at the two ends of the inlet. Since the computations needed to determine the additional current are too involved, they will be ignored at this time.

Manning's Roughness of Lewes and Rehoboth Canal

The main difficulty in the analytical evaluation of the surface fluctuation of a bay connected with the ocean through an inlet or a navigation canal is the assignment of a proper value of the Manning's n for the roughness. Not many references to this question are mentioned in the literature, and it was thought to be worthwhile to attempt to obtain the hydraulic roughness of the Lewes and Rehoboth Canal on the basis of the field investigation results of the summer of 1930 relative to the amounts of inflow and outflow transpiring during complete tidal cycles.

As indicated previously, the mean velocity through the canal can be expressed as

$$V = \sqrt{\frac{2grH}{\lambda L + r}}\sqrt{\frac{\Delta H}{H}} \tag{7}$$

where H is half of the tide range in ocean, r is the hydraulic radius of the canal, and λ is the coefficient of friction. The connection between λ and n is given by equation 15. The quantity ΔH is the difference between the levels of the waters of Delaware Bay and Rehoboth Bay. Observations have shown that during the period of field investigations the internal tides of the two bays, Rehoboth and Indian River, were insignificant. On the other hand, the bay waters showed a superelevation Δ_o with respect to msl. Analysis places this at the value Δ_o = 0.76 ft. Measuring from the instant that the ocean tide is rising from the mtl, during the inflow or the flood

$$\Delta H_i = H \sin \sigma t - \Delta_o \ , \ \sigma = 2\pi/T$$

and during the outflow or the ebb

$$\Delta H_o = \Delta_o - H \sin \sigma t$$

Let t_1 be the first instant when the waters of Rehoboth Bay and of Delaware Bay are at the same level. Then, the inflow during a tidal

cycle Q_i and the outflow Q_o are

$$Q_i = \int_{t_1}^{\frac{T}{2}-t_1} Vadt \text{ and } Q_o = \int_{\frac{T}{2}-t_1}^{T+t_1} Vadt$$

Introducing V from equation 7 and denoting the canal cross section during flood and ebb by a_1 and a_2, respectively, and the corresponding hydraulic radii by r_1 and r_2, one has

$$Q_i = \frac{a_1}{\sigma}\sqrt{\frac{2gr_1H}{\lambda L + r_1}}\, I_i$$

where

$$I_i = \int_{\sigma t_1}^{\pi-\sigma t_1} \sqrt{\frac{\Delta H_i}{H}}\, d\sigma t$$

and

$$Q_o = \frac{a_2}{\sigma}\sqrt{\frac{2gr_2H}{\lambda L + r_2}}\, I_o$$

where

$$I_o = \int_{\pi-\sigma t_1}^{2\pi+\sigma t_1} \sqrt{\frac{\Delta H_o}{H}}\, d\sigma t$$

The integrals I_i and I_o were evaluated numerically using the data: $\sigma t_1 = 0.38$ radian, $\Delta_o = 0.76$ ft, and $H = 2.05$ ft. It was found that $I_i = 1.44$ and $I_o = 3.45$;

then

$$0.69\,\frac{\sigma Q_i}{a_1} = \sqrt{\frac{2gr_1H}{\lambda_1 L + r_1}} \tag{97}$$

and

$$0.29 \frac{\sigma Q_o}{a_2} = \sqrt{\frac{2gr_2H}{\lambda_2L + r_2}} \tag{98}$$

With the values of the discharges known, i.e. $Q_i = 9.73 \times 10^6$ ft^3 and $Q_o = 16.1 \times 10^6$ ft^3 (see table 6), equations 97 and 98 can be used to evaluate two values of λ and then, through equation 15, the corresponding n values.

To complete the computation, the canal dimensions need to be established. The canal length was about 11.7 miles between the southern end of the canal and the northern end which connects with Delaware Bay via Broadkill River mouth. At some places the Lewes and Rehoboth Canal passes through marshland, a circumstance that prohibits an accurate description of an effective cross section from a hydraulic point of view. For the problem at hand it may be sufficient to assume a uniform trapezoidal cross section throughout the channel with a bottom width b_o and with the side embankments 1:m, and depth d. In a trapezoidal channel the area equals $(md + b_o)d$; the wetted perimeter, $2dm + b_o$. It is gathered from the IRIC report that the bottom width was $b_o = 50$ ft , whereas the water depth was 6 ft referred to mlw. Recalling that $\Delta_o = 0.76$ ft and $H = 2.05$ ft, the average depth in the channel was $d_1 = 9.4$ ft during high tide and $d_2 = 7.4$ ft during low tide. Assuming that $m = 2$, known as a good average value,

$$a_1 = 651 \text{ ft}^2$$
$$r_1 = 7.4 \text{ ft}$$

and

$$a_2 = 480 \text{ ft}^2$$
$$r_2 = 6.0 \text{ ft}$$

Introducing these values in equations 97 and 98 and the proper values of Q_i , Q_o , and L one finds finally that during ebb

$$\lambda = 6.6 \times 10^{-3}$$

and during flood

$$\lambda = 7.4 \times 10^{-3}$$

Correspondingly, during ebb

$$n = 0.020$$

and during flood

$$n = 0.022$$

Manning's Roughness of Indian River Inlet

The velocity measurements of 27 July 1948 are sufficiently complete for the determination of Manning's n of the inlet passage. The measurements were made at the location of the bridge, sta 20+00. The cross section of the inlet at this place had been divided into six parts and the velocities in them determined by current meters placed at 0.6-ft depths. Assuming that the distribution of velocities in a vertical line is in accordance with Manning's law, velocity readings at such depths would give the value of the mean velocities. The channel mean velocities as determined from the individual meters are entered in table 8. Also shown in this table are the time of measurement, the local tides, the tides at Atlantic City, and the magnitude of instantaneous cross sections.

It will be assumed that the range of tide at the ocean end of the inlet is similar to that at Atlantic City. On the day of the velocity measurements the range at Atlantic City was 3.30 ft from low to high and next 2.70 ft from high to low, and 3.00 ft from low to high. The mean, 3.00 ft, will be taken as the apparent ocean tide at the inlet during the day of the observations. Accordingly, $H = 1.5$ ft. It is preferable to consider the quantity $V/\sqrt{2gh}$ to study the variation of mean velocity with time. In this manner the validity of the velocity data can be extended to any ocean tide range. Making the reductions of the velocity data from table 8 on this basis, and using $H = 1.5$ ft, the results are shown in fig. 12.

Obviously, the velocity observed at the bridge should depend on the difference of the tides, H_2, of the ocean at the inlet and H_x at the bridge. The latter was observed and the former may be deduced from the

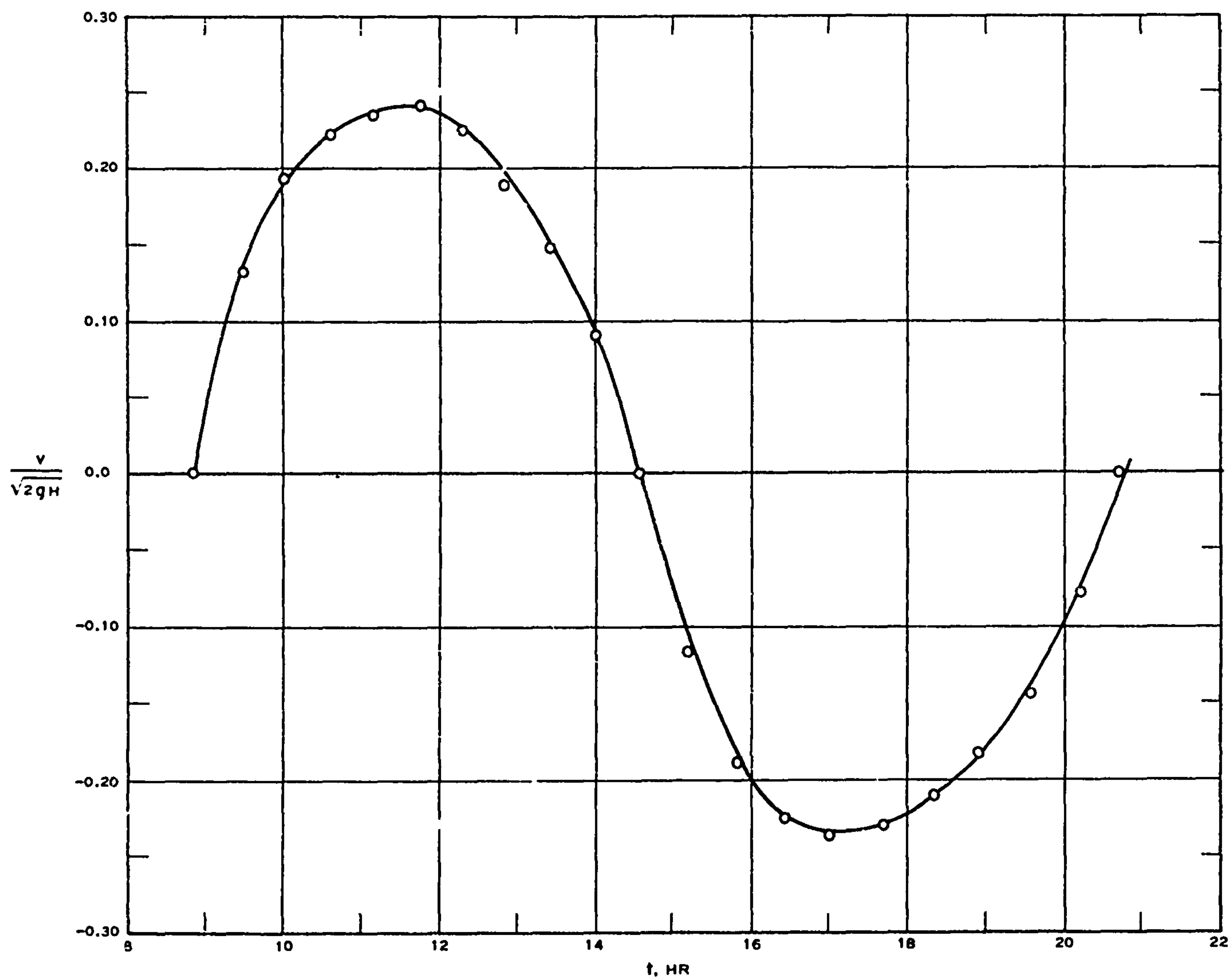

Fig. 12. Inlet channel mean velocity observed at the bridge, 27 July 1948

Atlantic City tide of the time of the velocity measurements. It will be assumed that the static water levels in the ocean and in the inlet at the bridge are at approximately the same elevation and that the range of tide at inlet ocean is nearly the same as the range at Atlantic City. Let H_{xr} and H_{2r} denote the tides at the inlet bridge and at Atlantic City, respectively, when referred to Indian River Bay datum. These are entered in the second and third columns of table 9. The average value of H_{xr} is 2.16 ft and the average value of H_{2r}, 2.42 ft. These differ but little from the corresponding mtl's of 2.07 and 2.35 ft. According to the assumption, the ocean tide H_x and inlet bridge tide H_2 with reference to the msl of the time are:

$$H_x = H_{xr} - 2.16$$

and

$$H_2 = H_{2r} - 2.42$$

These are entered in table 9. In the same table are also shown the values of h_x and h_2, representing, respectively, the ratios H_x/H and H_2/H with $H = 1.5$ ft. The variation of h_x and h_2 with time is shown graphically in fig. 13. Examining the velocity curve in fig. 12, it is seen that the slacks occur approximately at the times of hours 9, 14.5, and 21. At the times of the slacks the tide curves of ocean and bridge locale should intersect. This would be fulfilled if the ocean curve based on Atlantic City tide in fig. 13 were moved to the right a distance equivalent to 1/2 hr. The curve thus adjusted should now represent the ocean tide at the inlet. On this basis, the lag of tide between Atlantic City and the inlet is 1/2 hr.

The velocity formula of equation 7 can now be written as

$$V_m = \sqrt{\frac{2grH}{\lambda L_x + r}} \left(\sqrt{h_2 - h_x}\right)_m$$

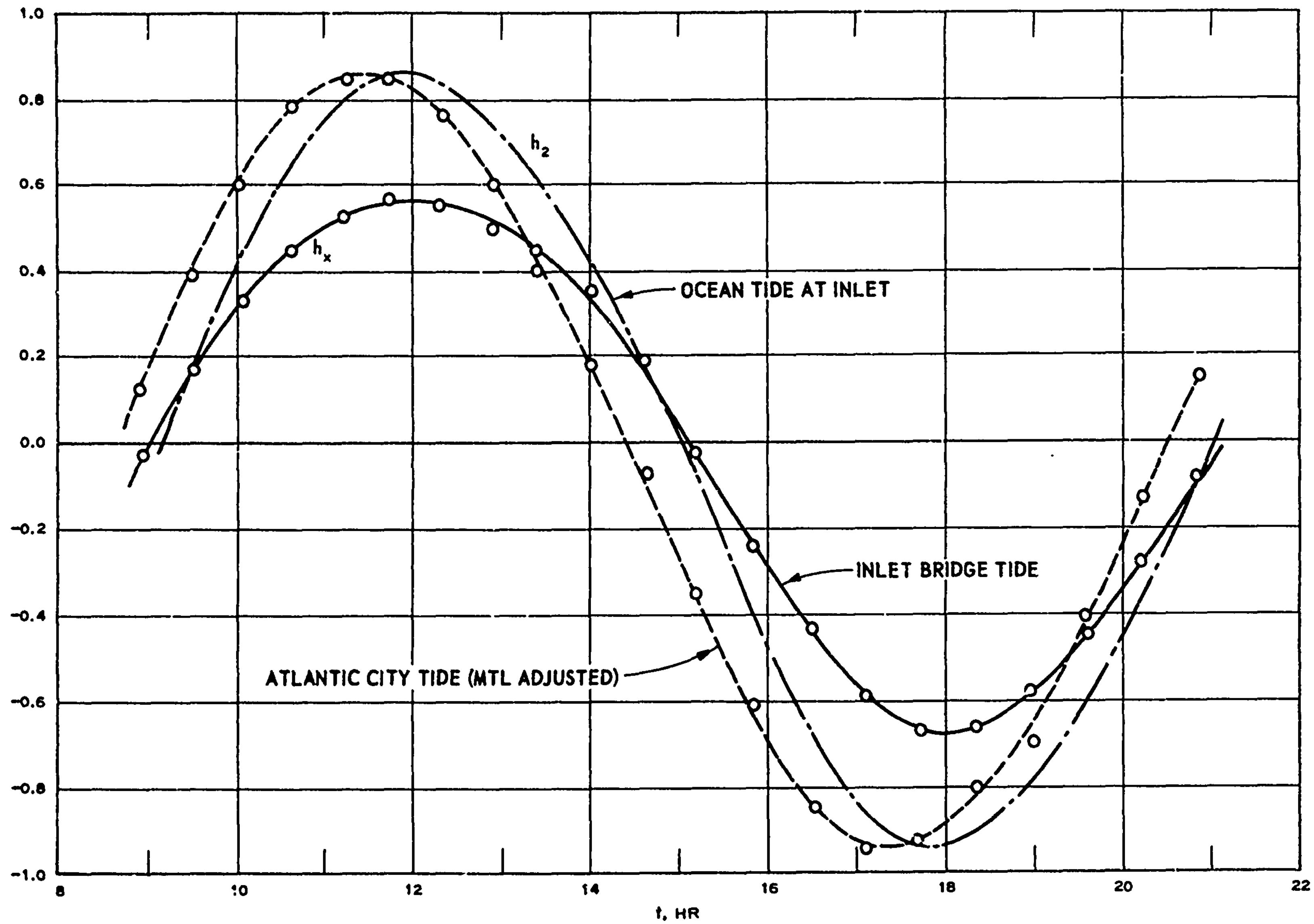

Fig. 13. Indian River Inlet Bridge and inlet ocean tides, 27 July 1948

where L_x is the distance between the bridge and the ocean end of the inlet, r is the average depth in the inlet between bridge and ocean, and V_m is the maximum velocity in mid-channel corresponding to the maximum value of the difference $h_2 - h_x$. The last equation can also be written as

$$\frac{V_m}{\sqrt{2gH}} = \sqrt{\frac{r}{\lambda L_x + r}} \left(\sqrt{h_2 - h_x} \right)_m \tag{99}$$

From fig. 12 the maximum current velocities observed during flood, time 11.7 hr, and during ebb, time 17.0 hr, appear to be of like absolute values; that is,

$$\frac{V_m}{\sqrt{2gH}} = 0.24$$

The differences in head for the corresponding times, see fig. 13, are also of like absolute values; that is,

$$\left(h_2 - h_x \right)_{max} \cong 0.27$$

At time 11.7 hr the cross section at the bridge, see table 8, is 10,740 ft^2, and at time 18.0 hr, it is 9,920 ft^2. As is expected, the mean of these extremes, 10,330 ft^2, is the same as the cross section corresponding to low-water slack time, or the area under msl.

Since the average cross section in the inlet between the bridge and the ocean is smaller than the cross section at the bridge, the maximum velocity for the inlet between the ocean and the bridge, a distance of L_x = 1430 ft, would be greater than the value shown above. The cross-sectional areas and the mean depths for various years are shown in table 7, and corresponding graphs are given in fig. 14. From the 1948 curve it is estimated that the average channel cross section, under msl and between sta 6+00 and 20+00, is 8390 ft^2. To arrive at this value, the stretch between the end stations was divided into 14 intervals, the areas at the interval midpoints were read from the plot, and the mean

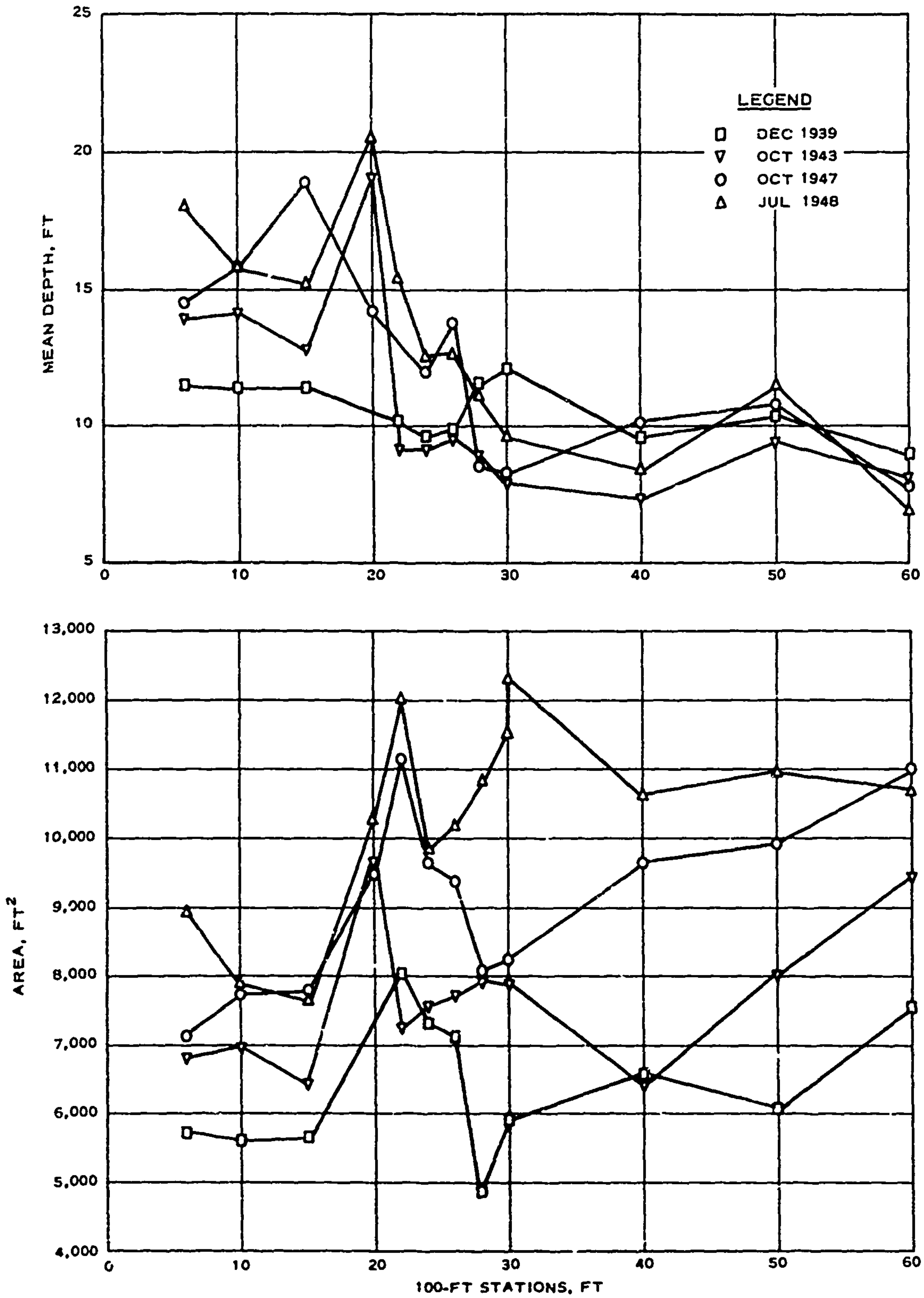

Fig. 14. Indian River Inlet mean depth and cross-sectional area at mtl

was established. In the same manner the channel average depth was estimated to be 16.8 ft. The ratio of the cross-sectional area at the bridge to the channel mean cross section is 1.23. Thus, the effective maximum current velocity is

$$\frac{V_m}{\sqrt{2gH}} = 0.24 \times 1.23 = 0.296$$

Taking this and remembering that

$$\left(h_2 - h_x\right)_{max} = 0.27$$

$$r = 16.8 \text{ ft}$$

$$L_x = 1430 \text{ ft}$$

equation 99 yields

$$\lambda = 246 \times 10^{-4}$$

and from equation 15

$$n = 0.046$$

As this is a roughness value considerably higher than the value ordinarily ascribed to water courses, it should be determined whether the ocean tide range was overestimated when basing it on the Atlantic City tide. The tide in the ocean north of North Jetty at the inlet mouth was observed. The observations as referred to mlw ocean datum, H_{2r} , are entered in the second column of table 10. Subtracting 2.16 from these gives the ocean tide as referred to the mean level of the tidal oscillations at the bridge. The value 2.16 ft was established previously, see table 9. The results, H_2 , are entered in the third column of table 10. The entries of the last column are h_2 , the ratio H_2/H , H being 1.5 ft. The data from the last column are plotted in fig. 15. The curve of

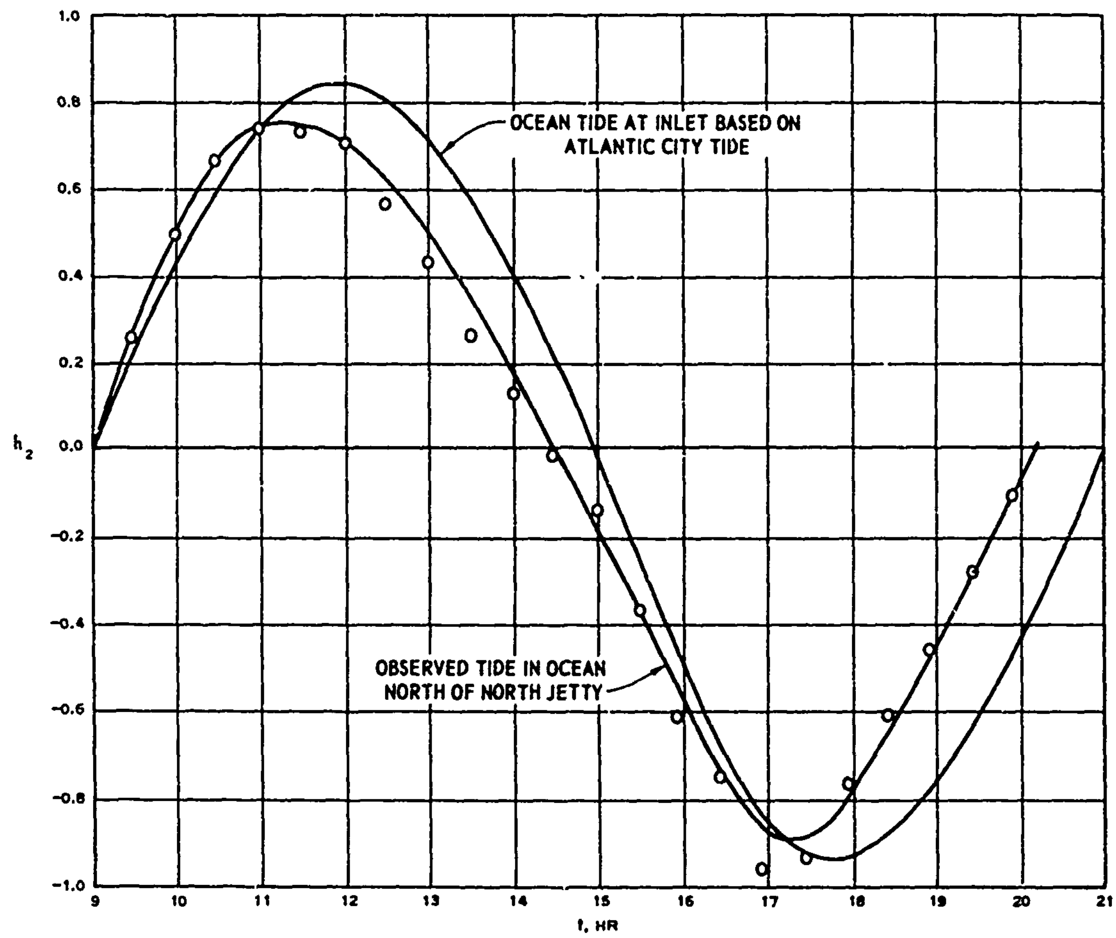

Fig. 15. Comparison of inlet ocean tides, observed and estimated

the ocean tide which was originally deduced from the Atlantic City data is reproduced therein from fig. 13. The two curves, one relating to Atlantic City and the other to the observations made at the inlet mouth, are closely aligned, showing that determination from the Atlantic City data is reliable.

In the above comparison the closer agreement is in regard to the range of tide and not in the temporal variation of tide for the two determinations. The form of the tide curve from the North Jetty observations seems to be an irregular one and therefore hardly useful for the present computation. Apparently, the determination of n made in the above should be acceptable. Since its value is large it would seem that other

causes are present to augment the resistance of the inlet channel. The interferences of the bridge piers should be considered if the tide gage for the bridge is placed west of the bridge. Fig. 16 shows that a severe movement of sand is connected with the channel depth changes, and these processes imply considerable energy dissipation, perhaps to such a degree as to augment the resistance in the channel.

Inertia Effect in Indian River Inlet

In the treatment of the problem of the preceding section it was assumed tacitly that the inertia effect in the flow through the inlet channel is ignorable. In the event that inertia effect is not ignorable, then at the times of slacks there would be a difference in head between the waters of the inlet at the bridge and of the ocean. It is now necessary to determine what this difference might be.

Expressing more precisely, the flow formula through a channel of uniform cross section would be

$$\frac{dV}{dt} = -gH\frac{dh}{dx} - \lambda\frac{V|V|}{r}$$

where the left term is the acceleration of waters and h is the surface displacement measured in terms of H, the semirange of tide in the ocean. Integrating between $x = 0$ and $x = L_x$:

$$\frac{L_x}{gH}\frac{dV}{dt} = -\left(h_x - h_2\right) - \frac{\lambda V|V|L_x}{grH}$$

Acceleration attains its maximum value at slack times when $V = 0$. The differential head due to inertia is

$$\begin{aligned}\left(h_2 - h_x\right)_0 &= \frac{L_x}{gH}\left(\frac{dV}{dt}\right)_0 \\ &= \frac{\sqrt{2}\,L_x}{\sqrt{gH}}\cdot\frac{1}{\sqrt{2gH}}\left(\frac{dV}{dt}\right)_0\end{aligned} \tag{100}$$

For the mean velocity of flow in the channel at the bridge, fig. 12 shows that

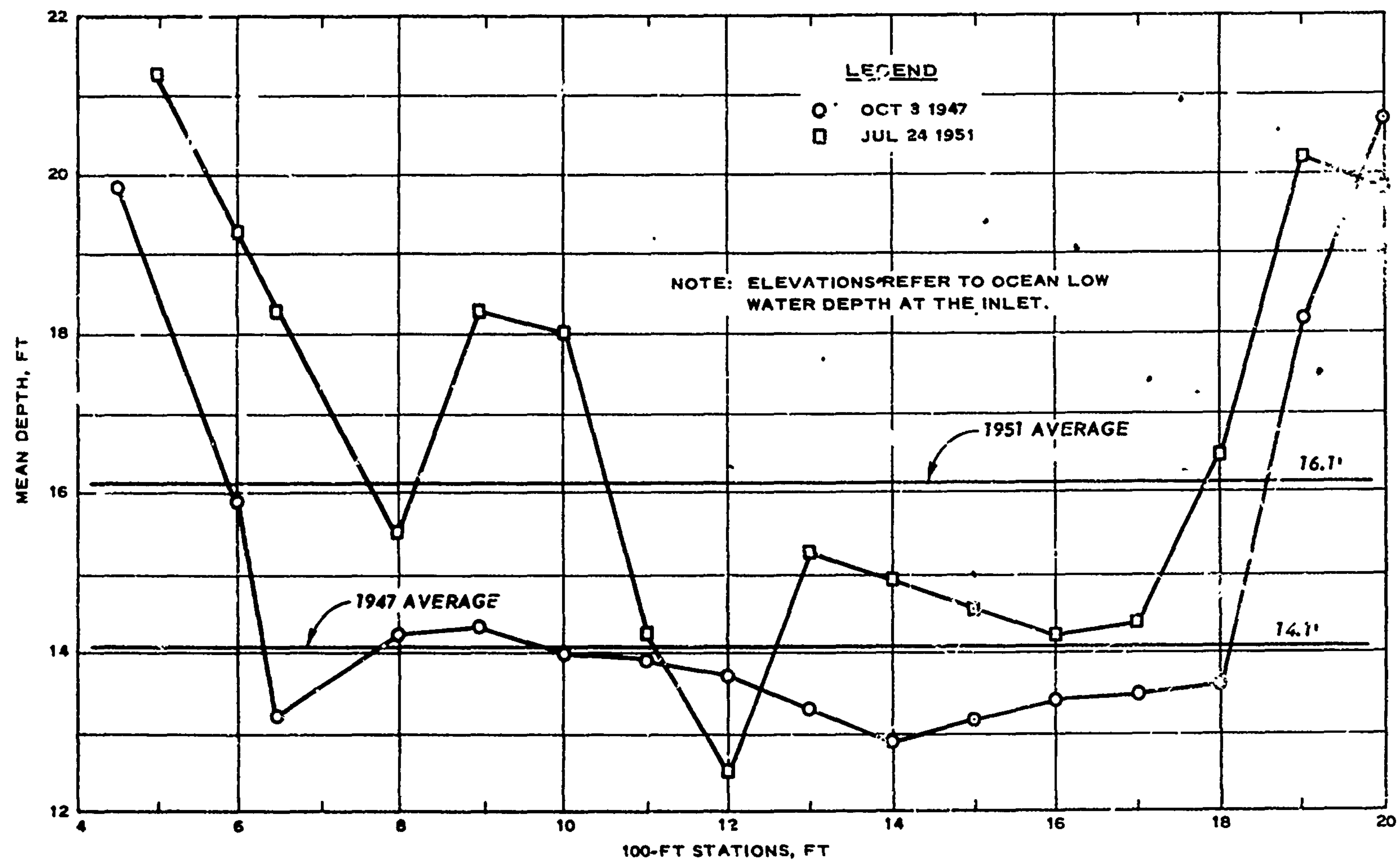

Fig. 16. Indian River Inlet channel mean depths

$$\frac{V}{\sqrt{2gH}} = 0.24 \sin \sigma t \ , \ \sigma = 2\pi/T$$

The corresponding expression for the channel between the bridge and the ocean would be 1.23 times as large, as explained in the previous section, and accordingly,

$$\left(\frac{d}{dt} \frac{V}{\sqrt{2gH}} \right)_0 = \frac{2\pi}{T} \times 0.24 \times 1.23 = 0.59 \frac{\pi}{T}$$

Substituting in equation 100,

$$\left(h_2 - h_x \right)_0 = 0.83 \frac{\pi}{T} \frac{L_x}{\sqrt{gH}}$$

and hence,

$$\left(H_2 - H_x \right)_0 = 0.83 \frac{\pi}{T} \frac{L_x}{\sqrt{gH}} H$$

With $L_x = 1430$ ft, $H = 1.5$, and $T = 4.47 \times 10^4$ sec, it follows that

$$\left(H_2 - H_x \right)_0 = 0.018 \text{ ft}$$

which is the maximum value for the head from inertia during a tidal period. It is indeed a small quantity and can be ignored.

Indian River Bay Tides

The formulas shown in the earlier sections were derived under a set of assumptions, and thus it is necessary to know how closely these assumptions are verified in the overall hydraulic conditions of the bay. One of the requirements of the assumptions was that there are no discharges into the basin from tributaries. There are tributaries emptying into the two bays, and by an analytical process it was found that the fall of water surface in 1929 and 1930 could be explained if the volume of water brought during a complete tidal period is about 15×10^6 ft^3. Since the velocity measurements at the inlet bridge, surveyed on 27 July 1948, indicate an

ebb outflow of nearly 400×10^6 ft^3 per tide, the discharges of rivers into Indian River Bay can be ignored. A second requirement was that the mean level of the water in the bay is in the same plane as the msl. The 1938-1939 observations indicate that the longtime mean superelevation was 0.15 ft in Indian River Bay and 0.08 ft in Rehoboth Bay (see table 5). Analytical considerations indicate that the actual superelevations may be lower, and it can be assumed that there is no significant superelevation. A final requirement is that the connection of the basin is with the ocean through a single inlet or with a multiple inlet system. This is not true for the Indian River Bay as there is an exchange of water between it and Rehoboth Bay through "The Ditches." However, since the volume of the water of the exchange is small in comparison with the volume of water traversing the inlet channel, one may determine the Indian River Bay tides assuming first that "The Ditches" are closed. Finally, these first evaluations can be corrected based on the hydraulics of "The Ditches."

The determination of the tides of the bay would be in reference to the three quantities: (1) the height of mean high tide in the bay, H_{1m} ; (2) the lag of tide, α ; and (3) the maximum current velocity, V_m , in the inlet passage. The formulas to evaluate these are:

$$H_{1m} = H \sin \tau \tag{54}$$

$$\alpha = \pi/2 - \tau \tag{55}$$

$$V_m = 2\pi\, C\, \frac{A}{a}\, \frac{H}{T} \sin \tau \tag{62}$$

In table 4 the dependence of $\sin \tau$ and C on the repletion coefficient K is shown. If, however, the ratio of the inlet cross section a to the surface area of the basin A is a small quantity, the use of the table can be dispensed with since in this case it suffices to write, for K less than 0.3 ,

$$\left.\begin{aligned} \sin \tau &= 1.14\,K \\ \text{and} \qquad C &= 0.812 \end{aligned}\right\} \tag{101}$$

These relations can be verified by referring to table 4 and fig. 6. As regards K, the coefficient of repletion, it is simply, taking $m = 1$,

$$K = \frac{T\sqrt{2gH}}{2\pi H}\sqrt{\frac{r}{\lambda L + r}}\,\frac{a}{A} \tag{12}$$

the relation between λ and n being

$$\sqrt{\lambda} = \frac{n\sqrt{2g}}{1.486\, r^{1/6}} \tag{15}$$

As can be seen from the entries in table 7, the inlet cross sections are far from uniform. It is necessary to normalize the cross sections to arrive at a prismatic channel of uniform cross section, its length the same as that of the actual channel, the hydraulic radius r equal to the averaged hydraulic radius of the actual channel, and its n value the same as that of the actual channel. Thus, the normalization establishes a new value for the cross section. Using the results of the cross-section survey of 1948 it is found that $r_m = 12.24$ ft and $a_m = 10{,}350$ ft^2 at msl. These values are obtained from the 1948 curves in fig. 14 after dividing the stretch between sta 6+00 and 60+00 into 20 equal segments and taking the mean of the areas or the depths for the midpoints of the segments. Application of equation 63 for normalization yields $a_{s1} = 0.944\, a_m$ and $a_{s2} = 0.924\, a_m$ for outflow, and thus the cross section of the normalized channel is $a = 9660$ ft^2. Recalling that the n for the inlet is 0.046 and $r = 12.24$ ft, equation 15 yields

$$\lambda = 243 \times 10^{-4}$$

The data for the determination of K from equation 12 are:

$$A = 420{,}000{,}000 \text{ ft}^2$$

$$a = 9660 \text{ ft}^2$$

$$L = 5500 \text{ ft}$$

$$r = 12.24 \text{ ft}$$

$$T = 12.42 \text{ hr or } 4.47 \times 10^4 \text{ sec}$$

$$H = 2.05 \text{ ft}$$

$$g = 32.2 \text{ ft/sec}^2$$

and

$$\lambda = 243 \times 10^{-4}$$

On the basis of these, equation 12 gives

$$K = 0.254$$

And using equation 101, it is found that

$$\sin \tau = 0.289$$

and

$$C = 0.812$$

Theoretically then, using equations 54, 55, and 62, the range of tide in the bay, $2H_{1m}$, is 1.19 ft, the lag of tide $\alpha = 1.276$ radians or 2.52 hr, and the maximum current velocity in the inlet is $V_m = 2.94$ ft/sec.

It is seen from table 5 that for the years 1948 and 1950 the mean range of tide at Oak Orchard was 0.93 ft and at Maple, 0.99 ft. The mean of these, 0.96 ft, can be taken to represent the range of tide realized at Indian River Bay. This mean is close to the theoretically computed range. In table 11 are shown longtime mean high- and low-water lunitidal intervals for different localities. It was argued in one of the preceding sections that the ocean tide at the inlet is about 1/2 hr later than the tide at Atlantic City. Adopting this and since lag of tide at Oak Orchard with reference to the tide at Atlantic City is about 3.33 hr, then the lag of the internal tide from the ocean tide at the inlet should be 2.83 hr, which is a value not far from the computed value. It was shown previously that the velocity survey of 27 July 1948 gives for the maximum current in the bridge area

$$V_m = 0.24\sqrt{2gH}$$

With $H = 2.05$ ft , the maximum current to be had in a cross section at the bridge would be $V_m = 2.76$ ft/sec . Since the ratio of the cross section at the bridge to the uniform cross section of the normalized channel of the computations is 10,330 to 9,660, then in the normalized channel $V_m = 2.95$ ft/sec , a value which is also close to the computed value.

The theoretical values of the quantities relating to the internal tides were carried out in the above, assuming that the water exchange through "The Ditches" is negligible. As there is some exchange of water, the next task should be to determine the effect of the exchange on the magnitude of the internal tides.

The Hydraulics of "The Ditches"

Table 11 shows the following data for 1948:

Location	HW Time, hr	Range of Tide, ft
Inlet Ocean	7.54	4.10
Oak Orchard	10.24	0.93
Dewey Beach	12.18	0.47

It will be assumed that the quantities observed at Oak Orchard and Dewey Beach are representative of Indian River Bay and Rehoboth Bay, respectively. On the basis of this data the courses of the tides in the three areas (i.e. the ocean, Indian River Bay, and Rehoboth Bay) would be as shown in fig. 17. During the entire time that the level of the water of the ocean continues to be higher than the level of the water of Indian River Bay, the respective positions of water levels in Indian River Bay and in Rehoboth Bay undergo a reversal. Thus, when an ebb of the inlet is filling Indian River Bay, there first is an inflow into Indian River Bay from Rehoboth Bay and next, an outflow. The balance of the discharges of these two flows has a bearing on the magnitude of the tides in Indian River Bay.

Let H_1 and H_o denote the tides in Indian River Bay and in Rehoboth Bay, respectively, and H_{1m} and H_{om} the maximum values of

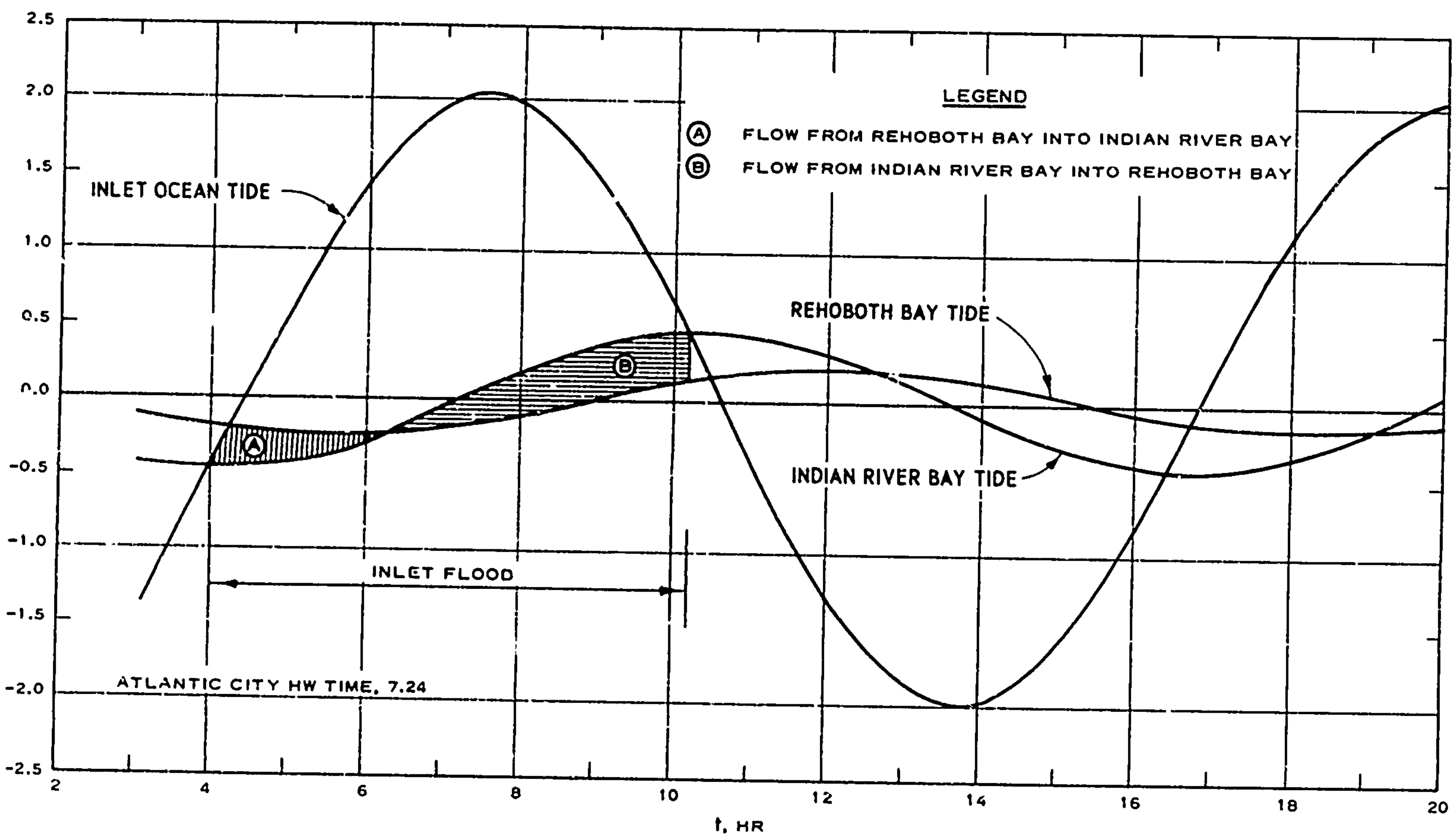

Fig. 17. Reversal of flow through "The Ditches" during inlet flood

these tides. Let time t be measured from the instant when the waters of Indian River Bay and the ocean are at the same level and the tide in the ocean is increasing. Let t_o denote the lag of tide in Rehoboth Bay with respect to the tide in Indian River Bay. The internal tides of the two bays are very nearly sinusoidal. Actually, there are higher harmonics, but these will be ignored. Accordingly, the expressions of the internal tides are:

$$\left.\begin{aligned} H_1 &= - H_{1m} \cos \theta \ , \ \theta = 2\pi t/T \\ H_o &= - H_{om} \cos (\theta - \tau_o) \ , \ \tau_o = 2\pi t_o/T \end{aligned}\right\} \qquad (102)$$

These are the inferences from the set. Ignoring the slight dependence of the lag of tide in Rehoboth Bay with respect to the tide in Indian River Bay upon the magnitudes of the internal tides, the value of the lag can be placed at $t_o = 2$ hr . The ratio of the tides in the two bays is constant, $H_{1m}/H_{om} = 2$.

Let t_1 be the time when the waters of the two bays are at the same level. At $t = t_1$ the flow through "The Ditches" reverses its direction. Place $\theta_1 = 2\pi t_1/T$. Relative time θ_1 is determined from the condition

$$\frac{H_o}{H_{1m}} = \frac{H_1}{H_{1m}}$$

which in view of equation 102 becomes

$$\cos \theta_1 = \frac{H_{om}}{H_{1m}} \cos (\theta_1 - \tau_o)$$

Lag t_o was shown to be 2 hr, and thus $\tau_o = 60°$. It was also shown that $H_{om}/H_{1m} = 1/2$; with this value the last relation reduces to

$$\cos \theta_1 = \frac{1}{4} \cos \theta_1 + \frac{\sqrt{3}}{4} \sin \theta_1$$

yielding

$$\theta_1 = 60^\circ$$

It is not necessary to consider the flows through the two "Ditches" separately, and for the purpose at hand one may as well take that the two "Ditches" can be replaced by a single one of cross section a which is the sum of the cross-sectional areas of the individual "Ditches." The velocity of inflow V_i into Indian River Bay is

$$V_i = k\sqrt{\frac{H_o}{H_{1m}} - \frac{H_1}{H_{1m}}}, \quad 0 \le \theta \le \theta_1$$

and the velocity of outflow is

$$V_o = k\sqrt{\frac{H_1}{H_{1m}} - \frac{H_o}{H_{1m}}}, \quad \theta_1 \le \theta \le \pi$$

Since a is the cross section, the volume of inflow is

$$Q_i = \frac{Tak}{2\pi}\int_0^{\theta_1}\sqrt{\frac{H_o}{H_{1m}} - \frac{H_1}{H_{1m}}}\, d\theta$$

and the volume of outflow is

$$Q_o = \frac{Tak}{2\pi}\int_{\theta_1}^{\pi}\sqrt{\frac{H_1}{H_{1m}} - \frac{H_o}{H_{1m}}}\, d\theta$$

Substituting from equation 102 and integrating, one has

$$Q_i = 0.63\,\frac{Tak}{2\pi} \tag{103}$$

and

$$Q_o = 1.58\,\frac{Tak}{2\pi} \tag{104}$$

Accordingly, the volume of outflow into Rehoboth Bay during the entire

time that Indian River Bay is being filled from the ocean is

$$Q_{oo} = 0.95 \frac{Tak}{2\pi} \tag{105}$$

For the determination of k, which is a quantity having dimensions (L/T^{-1}), proceed as follows. Place

$$k = C_D \sqrt{2gH_{1m}} \tag{106}$$

where C_D is a dimensionless number, a coefficient. Comparing the expression for the V_i velocity given above with equation 7 it will be inferred that

$$C_D = \sqrt{\frac{r}{\lambda L + r}} \tag{107}$$

The two "Ditches" were surveyed in 1935, and the cross-sectional proportions are shown in table 12. Combining the two "Ditches," the average value of r when the waters are at msl is 5.8 ft. The passages are regarded to be very inefficient for the exchange of water between the two bays. In accordance with this idea it is proper to assign to n the value of 0.10. With $r = 5.8$ ft and $n = 0.10$, equation 15 yields $\lambda = 0.161$. Appearance of the topography of the island separating the two "Ditches," see fig. 8, suggests that the channel length of "The Ditches" is about the same as the maximum width of the island, i.e. $L = 600$ ft. With $\lambda = 0.161$, $r = 5.8$ ft, and $L = 600$ ft, equation 107 yields

$$C_D = 0.24$$

and

$$k = 0.24 \sqrt{2gH_{1m}}$$

Inserting this in equation 105, the volume of outflow into Rehoboth Bay during the entire time that Indian River Bay is being filled from the ocean is

$$Q_{oo} = 0.23 \frac{aT}{2\pi} \sqrt{2gH_{1m}} \tag{108}$$

This applies independent of the magnitude of H_{1m} , provided the tide in Rehoboth Bay lags 2 hr behind the tide in Indian River Bay and the amplitude of the tide in Rehoboth Bay is half the amplitude of the tide in Indian River Bay.

The above treatment is introduced for the main purpose of formulating the correction needed in the evaluation of Indian River Bay tides. Although the flow through "The Ditches" reverses its direction during the time Indian River Bay is being filled by the flood of the inlet, for the correction it suffices to assume that the flow is in phase with the flow through the inlet and that the total flow is of the same magnitude as actually realized by the currents of "The Ditches." This is permissible. Denote now the assumed hypothetical flow through "The Ditches" as:

$$V_o = V'_{om} \sin \theta$$

This gives

$$Q_{oo} = \frac{Ta}{2\pi} \int_0^{\pi} V'_{om} \sin \theta \, d\theta$$

or

$$Q_{oo} = \frac{Ta}{\pi} V'_{om}$$

Comparing this with equation 108 yields

$$V'_{om} = 0.115 \sqrt{2gH_{1m}} \tag{109}$$

Indian River Bay Tides After Correction

In the presence of "The Ditches," the storage equation of Indian River Bay is

$$A \frac{dH_1}{dt} = a\ V - a_o\ V_o \qquad (110)$$

where H_1 is the elevation of the waters of Indian River Bay referred to msl, A is the surface area of the bay, V is the mean velocity of the inlet currents, V_o is the mean velocity of the currents of "The Ditches," a is the cross-sectional area of the inlet, and a_o is the cross-sectional area of "The Ditches." From above

$$A \frac{dH_1}{dt} = a\ V \left(1 - \frac{a_o}{a} \frac{V_o}{V}\right)$$

which, after using the transformation relating to equations 11, 12, and 13, will become

$$\frac{dh_1}{d\theta} = K\sqrt{h_2 - h_1}\left(1 - \frac{a_o V_o}{aV}\right),\ h_2 > h_1$$

The value of the expressions contained in the parentheses is close to unity, so that in forming the value it suffices to take the velocities separately as being sinusoidal. Consequently,

$$\frac{V_o}{V} = \frac{V'_{om} \sin\theta}{V_m \sin\theta} = \frac{V'_{om}}{V_m}$$

and the last differential equation simplifies to

$$\frac{dh_1}{d\theta} = K'\sqrt{h_2 - h_1},\ h_2 > h_1 \qquad (111)$$

where

$$K' = K\left(1 - \frac{a_o V'_{om}}{aV_m}\right)$$

The latter represents the correction in the coefficient of repletion K made necessary by the presence of "The Ditches."

The first estimate of H_{1m} was $H_{1m} = 0.59$ ft . Using equation 109, $V'_{om} = 0.74$ ft/sec . Also in the first estimate it was found that $V_m =$ 2.96 ft/sec . It is seen from table 12 that the combined area of the two "Ditches," surface waters at msl, is $a_o = 5322$ ft^2. It will be recalled that the inlet cross section after the normalization of the channel is $a = 9660$ ft^2. Hence,

$$\frac{a_o}{a}\frac{V_{om}}{V_m} = 0.138$$

and the corrected repletion coefficient is

$$K' = 0.86K$$

and

$$K' = 0.86 \times 0.254 = 0.218$$

Accordingly, one now finds that the range of tide in Indian River Bay is 1.02 ft instead of 1.19 ft and the lag of tide is 2.62 hr instead of 2.52 hr. These are quite close to the measured values of 1948 or 1950. As regards the maximum current velocity in the inlet, this now is 2.77 ft/sec instead of 2.95, and is an estimate lower than the observed value.

Erosion of Banks West of Highway Bridge

Among the numerous manifestations involving the movement of sand in Indian River Inlet, such as the formation of bars, depositions in the inlet channel, and the stability of dredged forms, attention perhaps should be given to the erosion of banks west of the highway bridge. Subsequent to the dredging in June 1938, it had been observed that the water widths experienced rapid and continued increases. The changes that did occur

between sta. 22+00 and 29+00 from October 1938 to April 1941 are shown in fig. 18. The underlying causes of this widening, a singular and remarkable phenomenon, were examined by Mr. Wicker. The following exposition is taken verbatim from Mr. Wicker's memorandum of 25 July 1939, addressed to the District Engineer, Philadelphia District.

> 1. In accordance with your instructions, I visited Indian River Inlet and made sufficient observations on July 17, 18, and 19, 1939 to ascertain the cause of the erosion of Indian River Inlet channel banks west of the highway bridge. I am convinced that wave attack is responsible.
>
> 2. The waves apparently occur only during flood current and the weaker parts of the ebb current. During my three days of observation, the maximum wave height above still water level was about one foot, the direction of approach was approximately normal to the bank throughout the embayment extending westward about 700 feet from the highway bridge, and the period of the waves was about 15 seconds, within ± 2 seconds. These waves do not appear to have a trough, but look and act like mounds of water moving rapidly toward the bank. This appearance, together with consideration of their period and height, suggests that the waves are of the translatory type. Translatory waves are often derived waves. In this case, it is believed that they are derived from the ocean waves, which probably impart impulses to the incoming flood current or check briefly the feebler portions of the ebb current. When the current is strongly ebb, it is believed that the energy of the ocean wave is rapidly sapped by the adverse current and there remains no energy to be dissipated by translatory waves.
>
> 3. Current velocities and directions in the embayment were measured. The situation on flood current is, briefly, as follows: The south embayment contains a counter-clockwise eddy which reverses the flow along the south bank from a point about 400 feet west of the bridge to the bridge. The maximum observed current velocity in the shoreward side of the eddy was about 7/10 feet per second, which occurred at a time when the main flood current thread had about 4 feet per second velocity. It is believed that 7/10 feet per second is not a scouring velocity. The embayment on the north has a clockwise eddy. Details were not observed, but it is believed that they would have been similar to those found on the south side. The ebb current moves past the embayments without creating any well-defined movement of water within them. The zone of moving water is sharply defined, as if there were bulkheads separating the waters of the embayments from the main threads of current.
>
> 4. The waves do their erosive work in the following manner: On the uprush, they strike the marsh mud or the scarp of the

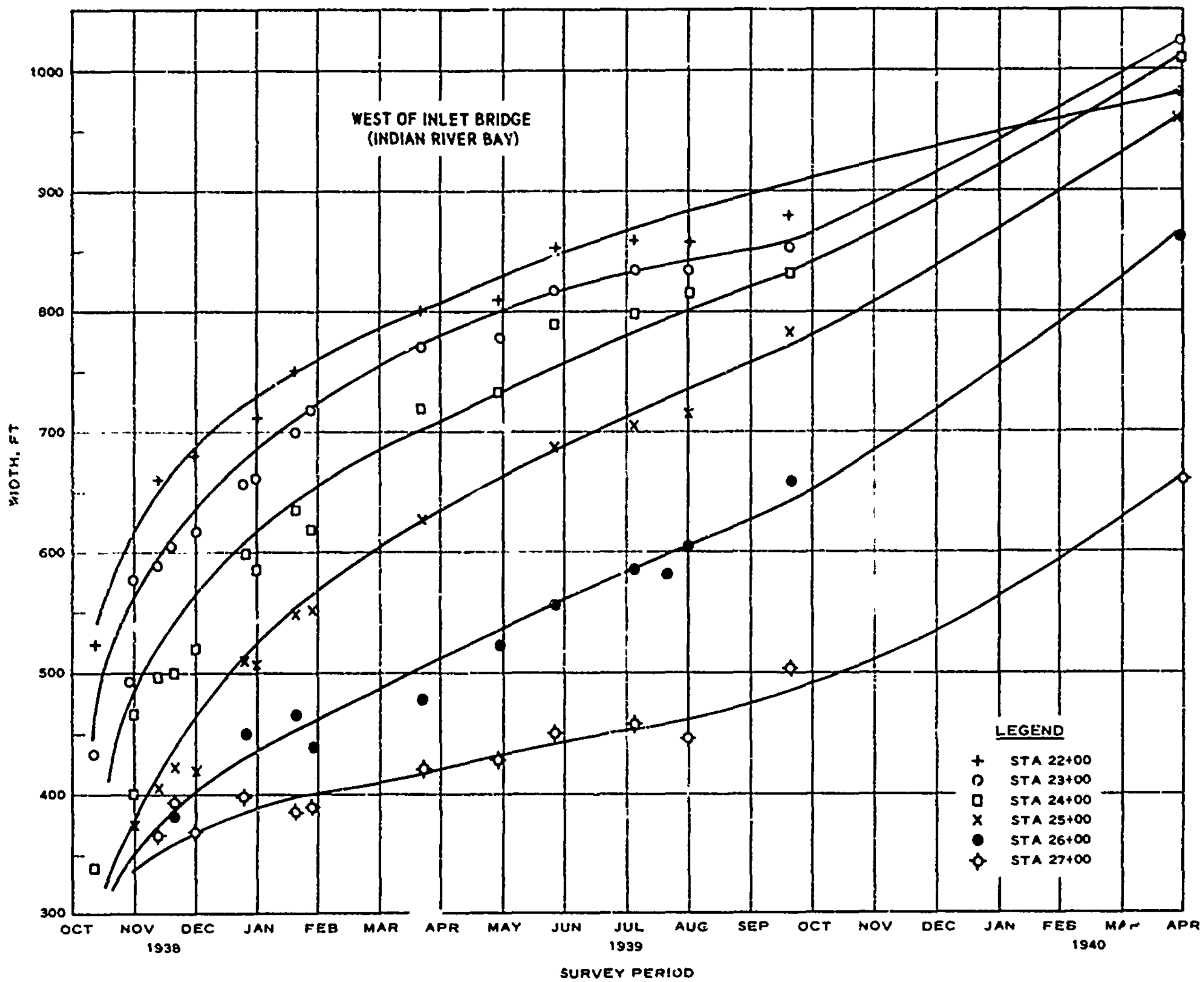

Fig. 18. Changes in high-water widths west of Inlet Bridge
(Indian River Bay)

spoil area, according to the stage of tide, with considerable force. The backwash is very strong, and sand moves steadily with it. The marsh mud is attacked, when the tide is sufficiently low, by removing the sand layer under it. When this has occurred, a piece of marsh mud held together by roots is broken off and eventually disintegrates. The material eroded is deposited on the bottom in the form of ridges. There were three such ridges apparent a short distance from the low water line, in depths of water from 0.5 to 1.5 feet. They were about four feet apart and about 6 or 8 inches high. Evidently the normal backwash cannot carry the sand that it erodes from the bank any farther. However, waves occurring during unusually low tides, or unusually large waves, could move the material farther channelwards, where it would be picked up by the inlet currents.

5. Erosion due to wave attack can be checked only by longitudinal works. In the case at hand, revetments appear to be an adequate form of protection. There is no evidence of settlement of the rip-rapping placed by the State, but it is being flanked on its westward end. The work should be done as soon as possible, for the erosion has now progressed to the spoil bank, and there is no reason for supposing it will be less rapid in the near future. Delay will result in the eventual deposition of a considerable quantity of sand in the channel.

It would appear that some of the facts singled out in Mr. Wicker's observations could be explained on the basis of turbulent expansion of the main current entering into an area with embayments on two sides. According to analysis, in an expanding jet there will be a lateral flow from two sides induced by turbulence and directed to the main current.[7] If the embayments are limited in area, the longitudinal flow at the edges of the expanding jet together with the lateral flow to the edges will enhance the formation of eddies in each embayment. The eddies were observed.

In a current moving between two embayments and experiencing lateral turbulent expansion, the current velocities decrease when moving laterally from the center line of the current toward embayment banks. Also, very likely the depth of water decreases from the center line away to the banks. These two conditions will cause the refraction of waves. If U is the current velocity and d the depth at a place, a point on the wave crest would be moving, in space, with velocity

$$w = \sqrt{gd} + U$$

Accordingly, when a solitary wave with crest originally normal to the inlet channel walls enters into the area with embayments, the point of the crest at the center moves away with greater velocity of propagation and points farther away from the center with lesser velocities. Consequently, the original wave breaks into two waves eventually, with alignments parallel to the banks and moving toward the bank. This also was observed.

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Table 1

Relation Between the Coefficient of Friction λ and Manning's n

r, ft	$\lambda \times 10^4$ n = 0.02	n = 0.03	n = 0.04	n = 0.05
5	68.04	153.26	272.58	425.18
10	54.02	121.66	216.38	337.82
15	47.22	106.29	189.06	295.15
20	42.93	96.63	171.87	268.30
30	37.45	84.33	150.06	234.09
40	34.05	76.67	136.42	212.87
50	31.62	71.18	126.56	197.68
60	29.72	66.93	119.02	185.78
70	28.25	63.60	113.21	176.62
80	27.02	60.82	108.16	169.00
90	25.97	58.49	104.04	162.31
100	25.05	56.43	100.00	156.75

Table 2

Tabular Values of the Coefficient of Repletion for $T = 12$ hr and $n = 0.02, 0.03, 0.04$, and 0.05

r	$(AK\sqrt{H/a}) \times 10^{-4}$					
	$L = 0.1 \times 10^4$	$L = 0.2 \times 10^4$	$L = 0.5 \times 10^4$	$L = 1.0 \times 10^4$	$L = 2.0 \times 10^4$	$L = 4.0 \times 10^4$
			$n = 0.02$			
5	3.697	2.859	1.974	1.443	1.112	0.741
10	4.557	3.825	2.867	2.180	1.606	1.160
15	4.810	4.364	3.437	2.708	2.042	1.495
20	5.004	4.611	3.830	3.110	2.397	1.780
30	5.293	5.018	4.637	3.722	2.952	2.254
40	5.295	5.120	4.618	4.053	3.209	2.626
60	5.383	5.327	4.938	4.537	3.909	3.195
80	5.426	5.338	5.101	4.770	4.262	3.596
100	5.449	5.383	5.195	4.925	4.500	3.899
			$n = 0.03$			
5	2.734	2.064	1.365	0.980	0.699	0.496
10	3.704	2.975	2.098	1.519	1.097	0.783
15	4.219	3.543	2.589	1.939	1.416	1.019
20	4.528	3.933	2.984	2.284	1.689	1.223
30	4.871	4.414	3.556	2.825	2.290	1.577
40	5.051	4.688	3.941	3.230	2.508	1.873
60	5.233	4.985	4.422	3.794	3.067	2.358
80	5.323	5.140	4.696	4.154	3.473	2.743
100	5.366	5.228	4.870	4.406	3.778	3.055
			$n = 0.04$			
5	2.171	1.598	1.038	0.740	0.525	0.385
10	3.101	2.389	1.604	1.158	0.828	0.589
15	3.732	3.264	2.040	1.495	1.077	0.765
20	4.044	3.344	2.397	1.780	1.292	0.927
30	4.504	3.900	2.948	2.245	1.662	1.134
40	4.761	4.252	3.328	2.626	1.971	1.441
60	5.037	4.665	3.909	3.191	2.473	1.846
80	5.175	4.894	4.262	3.596	2.866	2.180
100	5.261	5.035	4.500	3.894	3.182	2.460
			$n = 0.05$			
5	1.793	1.311	0.836	0.594	0.422	0.298
10	2.675	1.979	1.303	0.935	0.666	0.473
15	3.202	2.668	1.675	1.213	0.869	0.637
20	3.603	2.873	1.985	1.452	1.036	0.745
30	4.133	3.447	1.580	1.858	1.353	0.973
40	4.454	3.837	2.881	2.194	1.615	1.168
60	4.819	4.336	3.452	2.726	2.055	1.512
80	5.012	4.622	3.843	3.128	2.412	1.796
100	5.123	4.815	4.131	3.392	2.713	2.046

Note: H , L , and r are in feet.

Table 3

Parameters of the Fluctuation of Water Surface
in the Basin as a Function of K

K	a_1	a_3	b_3	cos τ	sin τ
0.1	0.9936	-0.0001	-0.0052	0.99327	0.115804
0.2	0.9745	-0.0004	-0.0106	0.97334	0.22934
0.3	0.9435	-0.0009	-0.0164	0.94086	0.33874
0.4	0.9020	-0.0017	-0.0220	0.89735	0.44137
0.5	0.8515	-0.0028	-0.0282	0.84425	0.53593
0.6	0.7942	-0.0043	-0.0347	0.78386	0.62091
0.7	0.7325	-0.0063	-0.0418	0.71856	0.69549
0.8	0.6689	-0.0089	-0.0495	0.65091	0.75917
0.9	0.5997	-0.0123	-0.0579	0.57732	0.81649
1.0	0.5451	-0.0165	-0.0664	0.51783	0.85551
1.2	0.4369	-0.0281	-0.0849	0.39949	0.91676
1.4	0.3489	-0.0448	-0.1038	0.30119	0.95357
1.6	0.2811	-0.0661	-0.1201	0.22449	0.97446
1.8	0.2294	-0.0910	-0.1327	0.16588	0.98614
2.0	0.1893	-0.1177	-0.1401	0.12160	0.99258
3.0	0.8830×10^{-1}	-0.2207	-0.1187	0.02953	0.99956
4.0	0.5032×10^{-1}	-0.2606	-0.0802	0.01037	0.99995
5.0	0.3232×10^{-1}	-0.2740	-0.0532	0.00575	0.99898
6.0	0.2249×10^{-1}	-0.2794	-0.0377	0.00363	0.99999
7.0	0.1653×10^{-1}	-0.2817	-0.0280	0.00256	1.0000
8.0	0.1266×10^{-1}	-0.2828	-0.0215	0.00192	1.0000
9.0	0.1001×10^{-1}	-0.2835	-0.0170	0.00150	1.0000
10	0.8105×10^{-2}	-0.2845	-0.0138	0.00119	1.0000
20	0.2026×10^{-2}	-0.2845	-0.0035	0.00030	1.0000
30	0.9007×10^{-3}	-0.2845	-0.0015	0.00013	1.0000
40	0.5066×10^{-3}	-0.2845	-0.0009	0.00008	1.0000
50	0.3242×10^{-3}	-0.2845	-0.0006	0.00005	1.0000
60	0.2252×10^{-3}	-0.2845	-0.0004	0.00004	1.0000
70	0.1654×10^{-3}	-0.2845	-0.0003	0.00003	1.0000
80	0.1267×10^{-3}	-0.2845	-0.0002	0.00002	1.0000
90	0.1001×10^{-3}	-0.2845	-0.0002	0.00001	1.0000
100	0.8105×10^{-4}	-0.2845	-0.0001	0.00001	1.0000

Table 4

Coefficient C in Tidal Prism Formula

and Range of Tide in Basin

K	sin τ	C	K	sin τ	C
0.1	0.1158	0.8106	4.0	0.9999	0.9993
0.2	0.2293	0.8116	5.0	0.9999	0.9994
0.3	0.3387	0.8128	6.0	1.0000	0.9997
0.4	0.4414	0.8153	7.0	1.0000	0.9997
0.5	0.5359	0.8184	8.0	1.0000	0.9998
0.6	0.6209	0.8225	9.0	1.0000	0.9998
0.7	0.6955	0.8288	10.0	1.0000	0.9998
0.8	0.7592	0.8344	20	1.0000	0.9998
0.9	0.8165	0.8427	30	1.0000	0.9999
1.0	0.8555	0.8522	40	1.0000	0.9999
1.2	0.9168	0.8751	50	1.0000	0.9999
1.4	0.9536	0.9016	60	1.0000	1.0000
1.6	0.9745	0.9267	70	1.0000	1.0000
1.8	0.9861	0.9484	80	1.0000	1.0000
2.0	0.9926	0.9650	90	1.0000	1.0000
3.0	0.9996	0.9950	100	1.0000	1.0000

Table 5

Longtime Mean Data of Tides in Indian River and Rehoboth Bays

Station	Year(s) of Observation	Duration of Observation months	Longtime Mean Values Range ft	Longtime Mean Values mtl ft	Longtime Mean Values mhw ft	Longtime Mean Values mlw ft	Staff El Referred to msl	Longtime Mean Values Referred to msl Range ft	Longtime Mean Values Referred to msl mtl ft	Longtime Mean Values Referred to msl mhw ft	Longtime Mean Values Referred to msl mlw ft
					Indian River Inlet and Bay						
Inlet Bridge	1938-1939	14	2.40	1.86	3.07	0.67	-1.90	2.40	-0.04	+1.17	-1.23
	1948	3	2.55	1.85	3.12	0.57	-1.89	2.55	-0.04	+1.23	-1.32
	1950	3-1/2	2.60	1.93	3.23	0.63	-1.89	2.60	+0.04	+1.34	-1.26
	Avg of 1948 and 1950	6-1/2	(2.58)	(1.89)	(3.18)	(0.60)	(-1.89)	(2.58)	(0.00)	(+1.29)	(-1.29)
Coast Guard Station	1946	1/2	1.17	+0.18	+0.77	-0.40	0.00	1.17	+0.18	+0.77	-0.40
Oak Orchard	1938-1939	14	0.56	2.01	2.29	1.73	-1.90	0.56	+0.11	+0.39	-0.17
	1948	2	0.93	2.12	2.59	1.66	-1.81	0.93	+0.31	+0.78	-0.15
	1950	3	0.93	2.23	2.70	1.77	-1.81	0.93	+0.42	+0.89	-0.04
	Avg of 1948 and 1950	5	(0.93)	(2.18)	(2.64)	(1.71)	(-1.81)	(0.93)	(+0.37)	(+0.83)	(-0.10)
Maple	1950	3	0.99	2.22	2.71	1.72	-1.84	0.99	+0.38	+0.87	-0.12
	Avg of 1948 and 1950*	3	(0.99)	(2.17)	(2.66)	(1.67)	(-1.84)	(0.99)	(+0.33)	(+0.82)	(-0.17)
					Rehoboth Bay						
Dewey Beach	1938-1939	14	0.28	1.92	2.06	1.78	-1.90	0.28	+0.02	+0.16	-0.12
	1946	1/2	0.28	+0.29	+0.43	+0.15	0.00	0.28	+0.29	+0.43	+0.15
	1948	1-1/2	0.47	2.13	2.37	1.90	-1.89	0.47	+0.24	+0.48	+0.01
Love Creek	1938-1939	14	0.21	1.97	2.08	1.87	-1.90	0.21	+0.07	+0.18	-0.03

* Adjusted by comparison with Oak Orchard observations to approximate average for 1948 and 1950.

Table 6

Inlet Commission Survey

Date	Discharge, ft³ Inflow	Discharge, ft³ Outflow	Duration, hr Inflow	Duration, hr Outflow
Indian River Inlet				
June 15-June 28	16,800,000	25,357,000	4.37	7.12
June 29-July 12	16,504,000	27,578,000	4.37	7.17
July 15-July 27	16,909,000	22,181,000	4.10	7.53
July 31-Aug 27	19,598,000	17,164,000	4.10	6.90
Aug 27-Sept 12	22,273,000	31,693,000	4.58	7.20
Average	18,417,000	24,795,000	4.30	7.18
Lewes and Rehoboth Canal				
June 15-June 27	8,182,000	13,731,000	4.72	6.95
June 28-July 9	9,158,000	15,455,000	4.40	7.32
July 12-July 26	9,588,000	16,753,000	4.60	7.22
July 27-Aug 20	10,063,000	15,718,000	4.90	6.89
Aug 22-Sept 12	11,654,000	18,918,000	4.55	7.14
Average	9,729,000	16,115,000	4.63	7.10

Net Outflow

Indian River Bay 6,378,000 ft^3 per tidal cycle

Rehoboth Bay 6,386,000 ft^3 per tidal cycle

Table 7

Indian River Inlet Channel Dimensions

Station	+2.0 Area, sq ft				Width at +2.0, ft				Mean Depth at +2.0, ft			
	1939	1943	1947	1948	1939	1943	1947	1948	1939	1943	1947	1948
6+00	5700	6800	7,110	8,960	493	493	493	493	11.56	13.79	14.42	18.17
10+00	5600	6980	7,750	7,850	493	493	493	493	11.36	14.16	15.72	15.92
15+00	5650	6420	--	7,670	494	502	--	502	11.44	12.79	--	15.28
20+00	--	9650	9,490	10,360	--	505	505	505	--	19.11	18.79	20.51
22+00	8040	7210	11,170	12,090	785	785	785	785	10.24	9.18	14.23	15.40
24+00	7335	7520	9,670	9,800	803	803	803	803	9.13	9.36	12.04	12.20
26+00	7165	7700	9,290	10,200	725	802	802	802	9.88	9.60	11.58	12.72
28+00	4870	7970	8,050	10,800	370	913	936	945	13.16	8.73	8.60	11.43
30+00	5960	7940	8,230	12,280	420	1000	1000	1350	14.19	7.94	8.23	9.10
40+00	6560	6440	9,620	10,650	720	890	945	1270	9.11	7.24	10.18	8.40
50+00	6060	8010	9,950	10,970	580	765	930	945	10.45	10.47	10.70	11.61
60+00	7590	9440	11,000	10,670	850	1330	1400	1550	8.92	7.10	7.86	6.88

Table 8

Indian River Inlet Velocities

Survey of 27 July 1948

Time, hr e.s.t.	Tide: Atlantic City	Tide: Inlet Bridge	Velocity ft/sec	Cross Section ft^2	Discharge ft^3/sec
8.95*	2.60	2.14	0.00	10,360	0
9.51	3.00	2.41	+1.29	10,470	+13,510
10.07	3.32	2.66	+1[illegible]	10,580	+20,100
10.64	3.53	2.83	+2.16	10,660	+23,030
11.20	3.69	2.95	+2.30	10,710	+24,630
11.76	3.70	3.00	+2.36	10,740	+25,350
12.32	3.56	2.98	+2.17	10,730	+23,280
12.88	3.32	2.91	+1.84	10,700	+19,690
13.45	3.01	2.82	+1.43	10,660	+15,240
14.01	2.69	2.68	+0.87	10,590	+9,215
14.57**	2.30	2.44	0.00	10,490	0
15.20	1.90	2.12	-1.17	10,350	-12,110
15.83	1.50	1.80	-1.90	10,210	-19,400
16.45	1.20	1.51	-2.23	10,080	-22,480
17.08	1.01	1.28	-2.36	9,975	-23,540
17.71	1.04	1.15	-2.30	9,920	-22,820
18.34	1.21	1.19	-2.10	9,940	-20,870
18.97	1.47	1.31	-1.82	9,990	-18,180
19.59	1.8[illegible]	1.50	-1.45	10,070	-14,600
20.22	2.24	1.75	-0.80	10,180	-8,145
20.85*	2.68	2.04	0.00	10,310	0

* Low-water slack.
** High-water slack.

Table 9

Established Dimensionless Tides at Inlet Bridge and at Inlet Ocean (H = 1.5 ft)

Time, hr e.s.t.	H_{xr}	H_{2r}	H_x	H_2	$h_x = \frac{H_x}{H}$	$h_2 = \frac{H_2}{H}$
8.95*	2.14	2.60	-0.02	0.18	-0.01	0.12
9.51	2.41	3.00	0.25	0.58	+0.17	0.39
10.07	2.66	3.32	0.50	0.90	0.33	0.60
10.64	2.83	3.53	0.67	1.11	0.45	0.79
11.20	2.95	3.69	0.79	1.27	0.53	0.85
11.76	3.00	3.70	0.84	1.28	0.56	0.85
12.32	2.98	3.56	0.82	1.14	0.55	0.76
12.88	2.91	3.32	0.75	0.90	0.50	0.60
13.45	2.82	3.01	0.66	0.60	0.44	0.40
14.01	2.68	2.69	0.52	0.27	0.35	0.18
14.57**	2.44	2.30	0.28	-0.12	0.19	-0.08
15.20	2.12	1.90	-0.04	-0.52	-0.02	-0.35
15.83	1.80	1.50	-0.36	-0.92	-0.24	-0.61
16.45	1.51	1.20	-0.65	-1.27	-0.43	-0.85
17.08	1.28	1.01	-0.88	-1.41	-0.59	-0.94
17.71	1.15	1.04	-1.01	-1.38	-0.67	-0.92
18.34	1.19	1.21	-0.97	-1.21	-0.65	-0.80
18.97	1.31	1.47	-0.85	-0.95	-0.57	-0.70
19.59	1.50	1.82	-0.66	-0.60	-0.44	-0.40
20.22	1.75	2.24	-0.41	-0.18	-0.27	-0.12
20.85*	2.04	2.68	-0.12	+0.26	-0.08	+0.17
Mean	2.16	2.42	--	--	--	--

* Low-water slack.
** High-water slack.

Table 10

Inlet Ocean Tide Observed North of North Jetty

27 July 1948

Time, hr e.s.t.	H_{2r} , ft	H_2 , ft	h_2
9.0	2.15	-0.01	-0.01
9.5	2.55	+0.39	+0.26
10.0	2.90	0.74	0.49
10.5	3.15	0.99	0.66
11.0	3.25	1.09	0.73
11.5	3.25	1.09	0.73
12.0	3.23	1.07	0.71
12.5	3.00	0.84	0.56
13.0	2.80	0.64	0.43
13.5	2.55	0.39	0.26
14.0	2.35	0.19	0.13
14.5	2.15	-0.01	-0.01
15.0	1.95	-0.21	-0.14
15.5	1.60	-0.56	-0.37
16.0	1.25	-0.91	-0.61
16.5	1.00	-1.16	-0.77
17.0	0.70	-1.46	-0.97
17.5	0.75	-1.41	-0.94
18.0	1.02	-1.14	-0.76
18.5	1.25	-0.91	-0.61
19.0	1.47	-0.69	-0.46
19.5	1.75	-0.41	-0.27
20.0	2.00	-0.16	-0.10

Table 11

Longtime Averages; High- and Low-Water Times and Ranges

Location	Time of High Water hr, e.s.t.	Time of Low Water hr, e.s.t.	Range of Tide ft
June, July, August, and September, 1948 Survey			
Atlantic City	7.24	1.12	4.08
Lewes	8.46	2.11	4.14
Indian River Inlet Bridge	7.76	1.47	2.55
Oak Orchard	10.24	4.65	0.93
Dewey Beach	12.18	6.43	0.47
January, February, March, and April, 1950 Survey			
Atlantic City	7.24	1.12	4.08
Indian River Inlet Bridge	7.89	1.60	2.60
Oak Orchard	10.53	4.63	0.93
Maple	10.93	5.50	0.99

Table 12

Cross-Sectional Dimensions of "The Ditches"

1935 Survey

Water Level* ft	Area, ft^2	Surface Width ft	Mean Depth ft
Little Ditch			
2.8	3150	420	7.5
2.0 (msl)	2816	416	6.8
0.0	2000	380	5.3
Big Ditch			
2.8	3620	900	4.0
2.0 (msl)	2506	894	2.8
0.0	1114	858	1.3

* Level referred to 1929 datum.